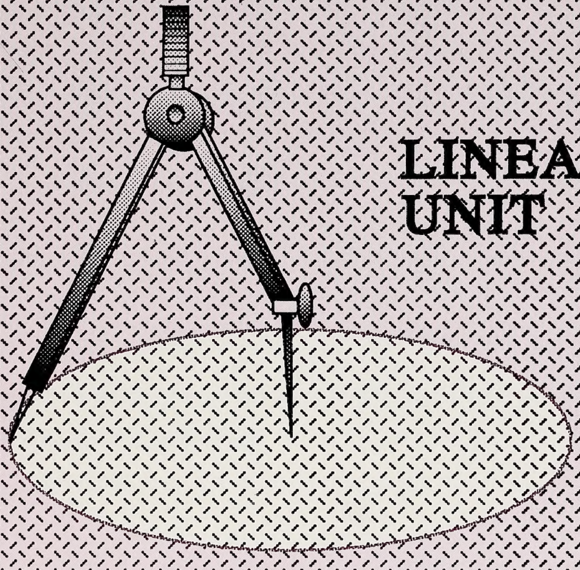
MATHEMATICS 23



LINEAR RELATIONS
UNIT 4

Distance Learning

Alberta EDUCATION

Welcome

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

Mathematics 23 Student Module Unit 4 Linear Relations Alberta Distance Learning Centre ISBN No. 0-7741-0790-1 *1992

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General Information

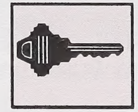
This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics (**Appendix B, etc.**).

Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

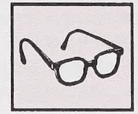
An explanation of what they mean is written beside each visual cue.



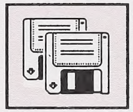
Key Idea
- flagging important ideas

Audiotape
- learning by listening to an audiotape

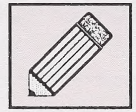
What You Already Know
- reviewing what you already know



Another View
- exploring different perspectives

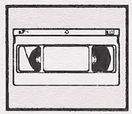


Computer Software
- learning by using computer software



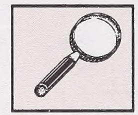
Review
- studying previous concepts

Solutions
- correcting the activities



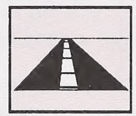
Videotape
- learning by viewing a videotape

Introduction
- introducing the unit

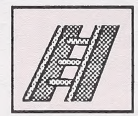


Extra Help
- providing additional study

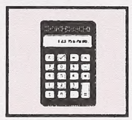
Print Pathway
- choosing a print alternative



What Lies Ahead
- previewing the unit

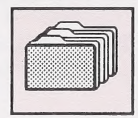


Extensions
- going on with the topic



Calculator
- using your calculator

Exploring the Topic
- actively learning new concepts



What You Have Learned
- summarizing what you have learned

Mathematics 23

Course Overview

Mathematics 23 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Powers and Radicals	10%
Unit 2 Algebra	12%
Unit 3 Mathematics of Finance	4%
Unit 4 Linear Relations	**12%**
Unit 5 Systems of Equations	16%
Unit 6 Geometry	16%
Unit 7 Trigonometry	16%
Unit 8 Statistics	14%
	100%

Unit Assessment

After completing the unit you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal your teacher will determine what this assessment will be. It may be

Unit assignment - 50%
Supervised unit test - 50%

Introduction to Linear Relations

This unit covers a topic dealing with Linear Relations. The topic contains explanations, examples, and activities to assist you in understanding linear relations. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

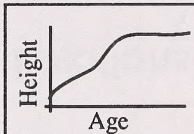
You can evaluate your understanding of each topic by working through the activities. Answers are found in the Solutions in **Appendix A**. In several cases there is more than one way to do the question.

Unit 4 Linear Relations

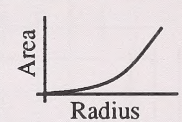
Contents at a Glance

Linear Relations

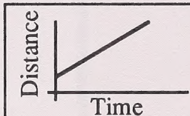
Relations can be represented in a visual manner and the idea of associating points with coordinates has been done for a long time. In the 14th century, **Nicole Oresme**, born in what is now France, suggested representing relations using graphs. Today, graphs are commonly used. The following are examples of graphs of relations.



Growth pattern for a person



Area of a circle compared to the radius



Distance travelled compared to time when speed is constant

Now you see that one of the graphs is a straight line. Linear relations are those relations that can be represented by points on a straight line and these are the subject of this unit.

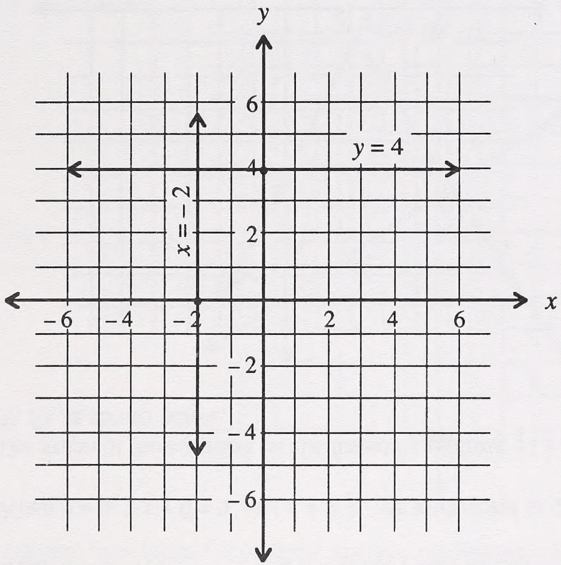
Topic 1:
Equations and
Graphs of
Linear Relations

Unit 4 Linear Relations

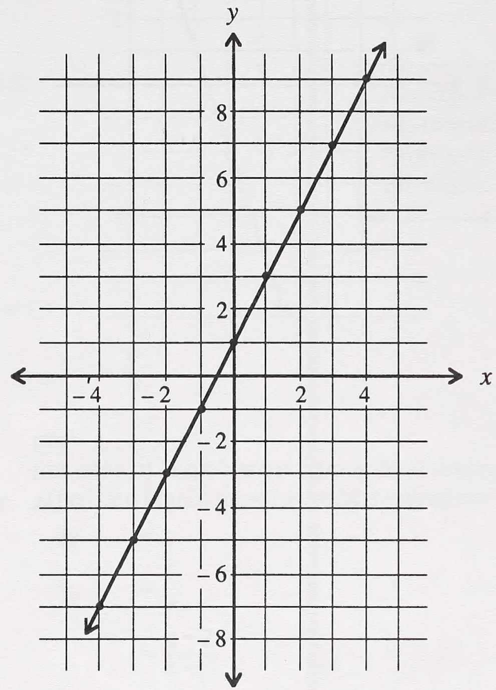
What You Already Know

Refresh your memory!

1. The graph of an equation of the form x = constant is a line parallel to the y-axis. The graph of an equation of the form y = constant is a line parallel to the x-axis. For example, the graphs of $x = -2$ and $y = 4$ are shown below.



2. A linear equation can be graphed by plotting a set of ordered pairs. For example, the graph of $y = 2x + 1$ can be found by plotting $(-4, -7)$, $(-3, -5)$, $(-2, -3)$, $(-1, -1)$, $(0, 1)$, $(1, 3)$, $(2, 5)$, $(3, 7)$, $(4, 9)$.



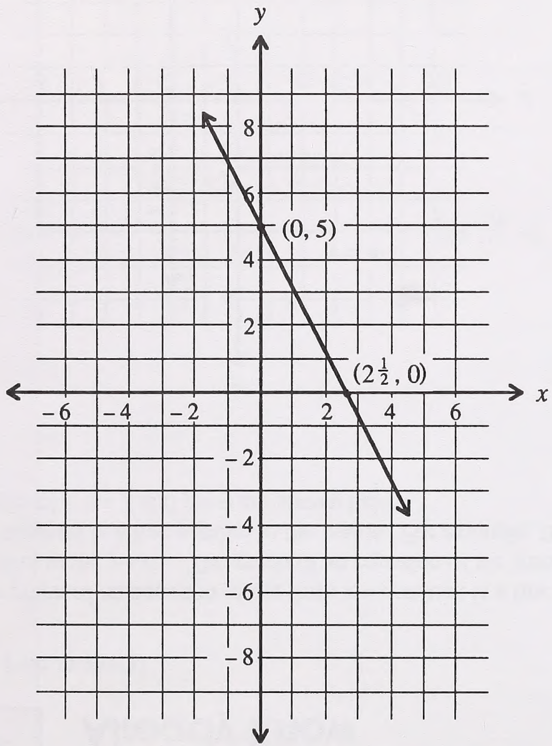
3. In a linear equation, the variables are to the first power. For example, $2x + 3y = 6$ and $y = -2x + 1$ are linear equations, but $4x^2 + y = 9$ and $y = x^2 + 4$ are not linear.

4. A linear equation can be graphed by plotting the x-intercept and the y-intercept. For example, $2x + y = 5$.

When $x = 0$, $2(0) + y = 5$, and $y = 5$, the y-intercept is 5.

When $y = 0$, $2x + 0 = 5$, and $x = 2\frac{1}{2}$, the x-intercept is $2\frac{1}{2}$.

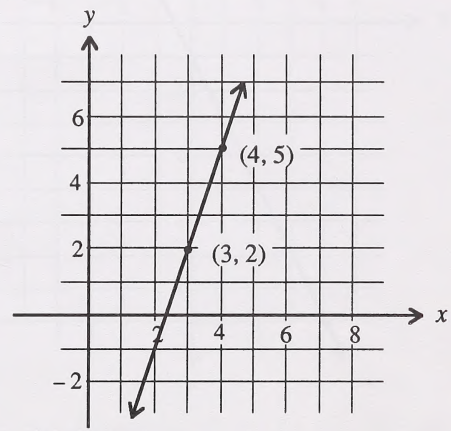
The graph of the equation can be found by plotting $\left(2\frac{1}{2}, 0\right)$ and (0, 5), as shown below.



5. The **slope** of a line can be found from the coordinates of two points on the line except when the points have the same x-coordinate. You can think of the slope as the "rise" over the "run". For example, the slope of the line through (3, 2) and (4, 5) can be found as follows.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5-2}{4-3} \\ &= \frac{3}{1} \\ &= 3 \end{aligned}$$

When two points show the same x-coordinate, both are on a line parallel to the y-axis. The slope is undefined for such a line.



6. Parallel lines have the same slope.
   For example, the graphs of $y = 3x + 5$ and $y = 3x - 1$ are parallel lines since they both have a slope of 3.

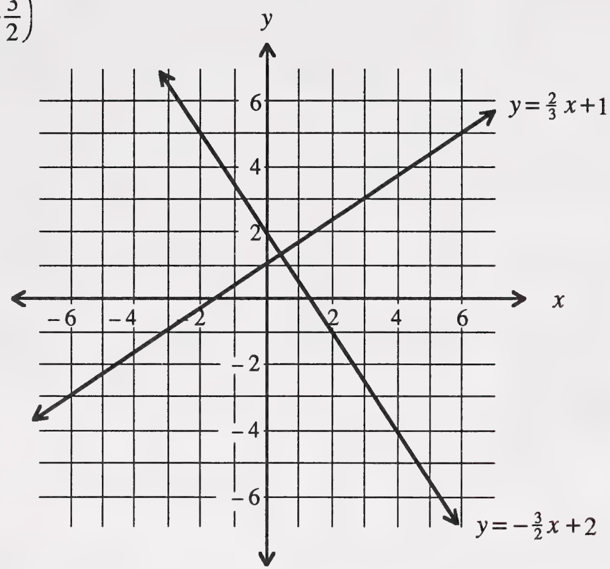
7. When both a line and its perpendicular have non-zero slopes, their slopes are related by the formula,

   $$m_1 \times m_2 = -1.$$

   For example, the graphs of $y = \frac{2}{3}x + 1$ and $y = -\frac{3}{2}x + 2$ are perpendicular.

   The slope of one graph is $\frac{2}{3}$ and the slope of the other is $-\frac{3}{2}$.

   $$\begin{aligned} m_1 \times m_2 &= \frac{2}{3} \times \left(-\frac{3}{2}\right) \\ &= -1 \end{aligned}$$



Now that you have looked at material that you studied previously, go to the **Review** to confirm your understanding of this material.

When one equation is in the $y = mx + b$ form, the coefficient of x gives the slope.

When one of the slopes is zero, the formula does not "work." When one of the factors is zero, the product cannot be –1.

Review

Try the following review questions.
Graph paper is provided in **Appendix B**.

1. Which of the following is a linear equation?

   a. $3x^2 + y = 6$ b. $y = x^4 + 3$
   c. $9x - y = 18$ d. $x + 2y^3 = 3$

2. Graph the equation $y = \frac{1}{2}x - 3$ by plotting five ordered pairs for the relationship.

3. Graph the equations $x = -1$, $x = 5$, and $y = 3$. Draw them all on the same graph.

4. Use the x-intercept and the y-intercept to graph the equation $y = \frac{1}{2}x + 3$.

5. What is the slope of the line passing through the points $(-5, 3)$ and $(4, -15)$?

6. Which equation has a graph parallel to the graph of $x - 3y = -3$?

   a. $x + 3y = -3$ b. $4x - 12y = -3$
   c. $y = -3x$ d. none of the above

7. Which equation has a graph perpendicular to the graph of $y = \frac{3}{4}x + 10$?

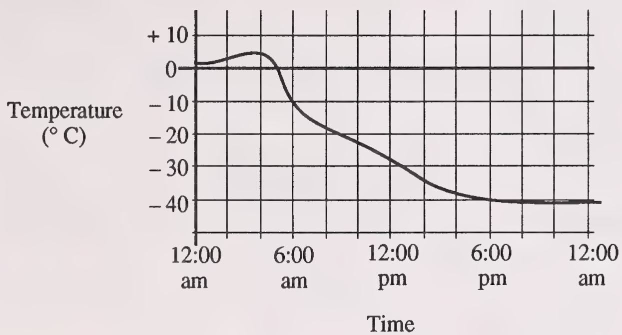
   a. $y = -\frac{4}{3}x + 4$ b. $y = \frac{4}{3}x + 10$
   c. $y = \frac{3}{4}x + 2$ d. none of the above

For the **Review** solutions, turn to **Appendix A**.

Topic 1 Equations and Graphs of Linear Relations

Introduction

When a graph is irregular, you need a complete table of values to draw it. Take for example, the graph showing the temperatures over the course of the whole day of January 31, 1989. You may remember that day for the worst January storm on record in Alberta. For a certain location, the graph looked as follows.



Many temperature readings were needed to make the graph. In contrast, for linear relations, both the graph can be drawn and the equation can be written by just knowing two points or one point and the slope. You will find out how in this topic.

What Lies Ahead

Throughout this topic you will learn to

1. write the equation and draw the graph of a linear relation given the slope and the y-intercept of the graph of the relation
2. write the equation and draw the graph of a linear relation given one point and the slope of the graph of the relation
3. write the equation and draw the graph of a linear relation given two points that are on the graph of the relation
4. write the equation and draw the graph of a linear relation given one point, and the equation of a line that is parallel or perpendicular to the required linear relation

Exploring Topic 1

Write the equation and draw the graph of a linear relation given the slope and the y-intercept of the graph of the relation.

When the slope and the y-intercept of the graph of a relation is given, you can always write the equation for the relation in the form

$$y = mx + b,$$

where m is the slope and b is the y-intercept. This is called the slope y-intercept form.

Example 1

Write the equation for the linear relation having a graph with a slope of $\frac{2}{3}$ and a y-intercept of 4.

Solution:

$$y = \frac{2}{3}x + 4$$

The coefficient of x is the slope and the constant term gives the y-intercept.

Example 2

A graph of a linear relation has a slope of -5 and a y-intercept of $\frac{3}{4}$. Give the equation for the relation.

Solution:

$$y = -5x + \frac{3}{4}$$

The general form of linear relations is $Ax + By + C = 0$. If C is subtracted from both sides, you get $Ax + By = -C$. This is how linear relations are often expressed.

A, B, and C are constant.

The equations $2x + 3y = 5$, $12x + 7y = 10$ and $3x - 2y = 10$ are in this form.

The equation $Ax + By = -C$ is said to be reduced when A, B, and $-C$ have no common factor. For example, $2x + 3y = 4$ is reduced, whereas $6x + 9y = 12$ is not.

The next example shows how you can write the equation in the $Ax + By = -C$ form when you are given the slope and y-intercept.

Example 3

A graph of a linear relation has a slope of $\frac{2}{5}$ and a y-intercept of 2. Give the equation for the relation in the $Ax + By = -C$ form.

Solution:

$$y = \frac{2}{5}x + 2$$

Make the term containing the x the first term on the left side. Its sign changes.

$$-\frac{2}{5}x + y = 2$$

Clear the fractions. Multiply through by 5.

$$-2x + 5y = 10$$

If the first term is negative, multiply through by –1. This changes the signs of all the terms.

$$2x - 5y = -10$$

The next example shows a situation in which the y-intercept, as well as the slope, is a fraction.

Example 4

A graph of a linear relation has a slope of $\frac{3}{4}$ and a y-intercept of $\frac{1}{6}$. Give the equation for the relation in the $Ax + By = -C$ form.

Solution:

$$y = \frac{3}{4}x + \frac{1}{6}$$

Make the term containing the x the first term on the left side.

$$-\frac{3}{4}x + y = \frac{1}{6}$$

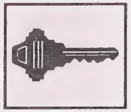
Clear the fractions. Multiply through by the lowest common denominator. Here the lowest common denominator is 12.

$$-9x + 12y = 2$$

Multiply through by –1 to make the first term positive. The signs of all the terms change.

$$9x - 12y = -2$$

The examples show that extra steps are involved in going to the $Ax + By = C$ form. Unless the context requires, display the equation in the $y = mx + b$ form.



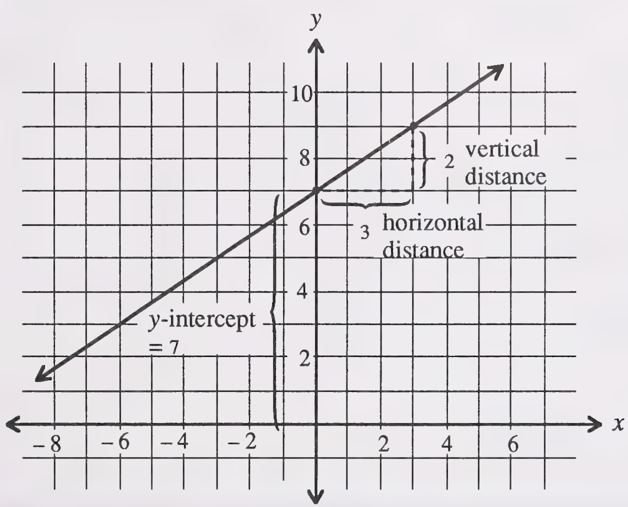
You can draw the graph from just the slope and the y-intercept. You proceed by plotting the y-intercept. Express the slope as a fraction. From the y-intercept, move horizontally according to the denominator and vertically according to the numerator. This takes you to a second point. Draw a line through this point and the y-intercept.

This method of graphing is used in the next examples.

Example 5

Draw the graph of the linear relation having a y-intercept of 7 and a slope of $\frac{2}{3}$.

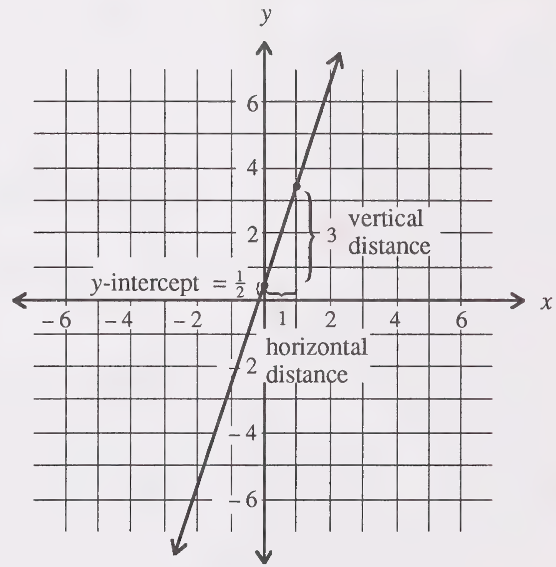
Solution:



Example 6

The graph of a linear relation has a slope of 3 and a y-intercept of $\frac{1}{2}$. Draw the graph.

Solution:

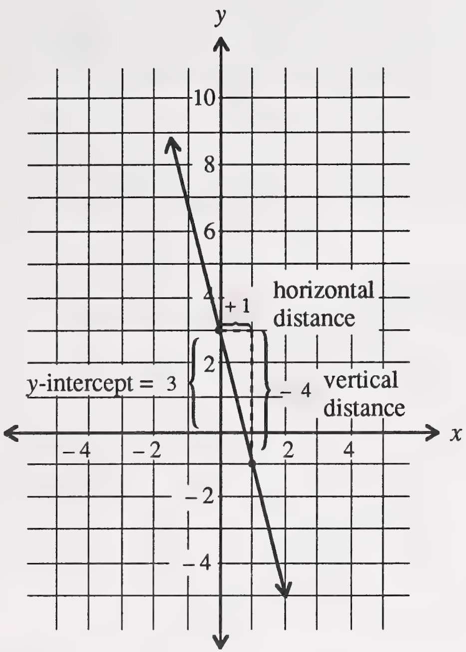


Note: As a fraction, the slope is $\frac{3}{1}$.

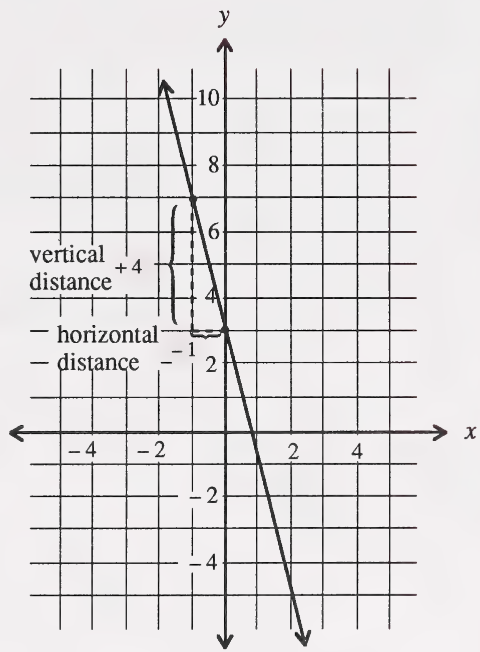
Example 7

Draw the graph of a linear relation that has a slope of – 4 and a *y*-intercept of 3.

Solution:



OR



As a fraction, the slope is $\frac{-4}{1}$ or $\frac{4}{-1}$.

The graphs are the same whether you consider the "rise" negative with the "run" positive or you consider the "rise" positive with the "run" negative.

Now try to write the equations and draw the graphs for the following questions. Graph paper is provided in **Appendix B**.

Do the even-numbered questions.

In the following five questions, the slope and y-intercept are given for particular linear relations. In each question, write the associated equation (in the $y = mx + b$ form).

1. The slope is 5 and the y-intercept is 4.

2. The slope is $-\frac{2}{3}$ and the y-intercept is -49.

3. The slope is 25 and the y-intercept is 0.

4. The slope is 0 and the y-intercept is 19.

5. The slope is -4 and the y-intercept is 3.

6. The graph of a linear relation has a slope of $\frac{1}{5}$ and a y-intercept of $-\frac{1}{10}$. Write the $Ax + By = -C$ form of the equation for this relation.

In the following four questions, the slope and y-intercept are given for particular linear relations. In each question, draw the associated graph.

7. The slope is $\frac{3}{5}$ and the y-intercept is 5.

8. The slope is -3 and the y-intercept is 5.

9. The slope is 0 and the y-intercept is -1.

10. The slope is $\frac{-5}{7}$ and the y-intercept is 0.

In two questions, the slope and y-intercept are again given for linear relations. In these questions draw the graphs and write the equations that represent the relations.

11. The slope is $-1\frac{1}{3}$ and the y-intercept is 1.

12. The slope is 2 and the y-intercept is $-2\frac{1}{2}$.

For solutions to **Activity 1**, turn to **Appendix A**, **Topic 1**.

Activity 2

Write the equation and draw the graph of a linear relation given one point and the slope of the graph of the relation.

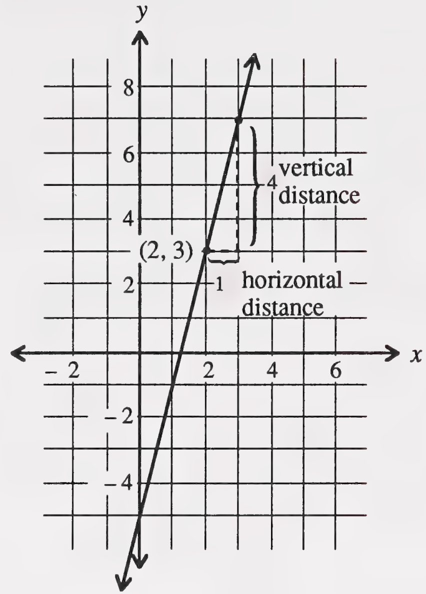
The technique of drawing the graph of a linear relation given the slope and a point other than the *y*-intercept is similar to the graphing used in the previous activity. Again, the given point is plotted and then the slope is used to plot a second point. Drawing a line through the points completes the graph.

Look at the examples.

Example 8

Draw the graph of a linear relation that has a slope of 4 and contains the point, (2, 3).

Solution:

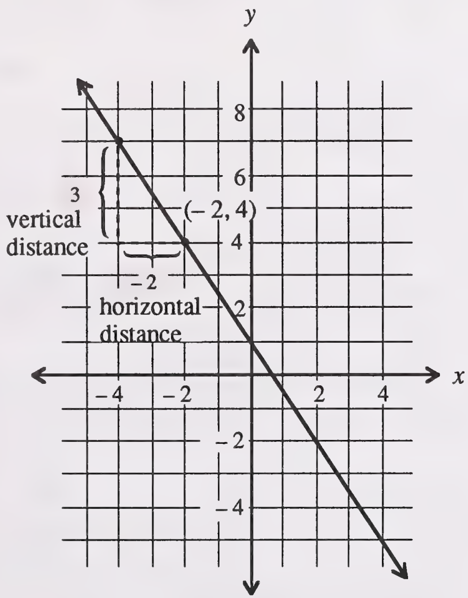


As a fraction, the slope is $\frac{4}{1}$.

Example 9

Draw the graph of a linear relation that has a slope of $\frac{-3}{2}$ and contains the point (– 2, 4).

Solution:



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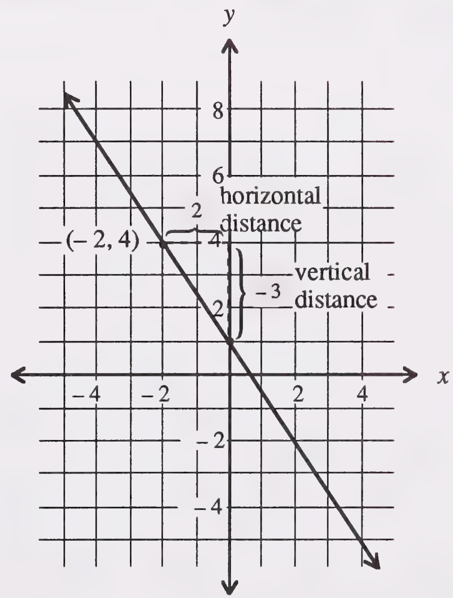


What about writing the equation when you have the slope and a point? Then you can write the equation in the form $y = mx + b$. You have values for x and y from the coordinates of the point, and m is given. This allows you to solve for b and then to write the specific equation. This is demonstrated in the next example.

The slope can be represented by $\frac{-3}{2}$ or $\frac{3}{-2}$.

Remember the graphs are the same regardless of whether you consider the rise positive with the run negative, or you consider the rise negative with the run positive. Generally, though, it is easier to consider the run positive with the rise taking the sign of the slope. This approach is used in the graph in the righthand column.

Example 10

Write the equation of a linear relation whose graph has a slope of 4 and contains the point (2, 3).

Solution:

The coordinates of the point tell you that when $x = 2, y = 3$.
These values can be substituted with the equation $y = mx + b$, where $m = 4$.

$$\begin{aligned} 3 &= 4(2)+b \\ 3 &= 8+b \\ b &= 3-8 \\ &= -5 \\ b &= -5 \text{ and } m=4 \end{aligned}$$

The equation for the relation is $y = 4x - 5$.

Example 11

Write the equation of a linear relation whose graph has a slope of $-\frac{3}{2}$ and contains the point $(-2, 4)$.

Solution:

From $(-2, 4)$, you can see that when $x = -2, y = 4$.

When you substitute these values and the value $m = -\frac{3}{2}$ into $y = mx + b$, you obtain

$$\begin{aligned} 4 &= \left(\frac{-3}{2}\right)(-2)+b \\ 4 &= 3+b \\ b &= 4-3 \\ &= 1 \end{aligned}$$

The equation for the relation is

$$y = \frac{-3}{2}x+1.$$

Example 12

Write the equation of a linear relation whose graph has a slope of 5 and contains the point (9, 45).

Solution:

$m = 5$
The coordinates of (9, 45) indicate that in the substitution, $x = 9$ and $y = 45$ in $y = mx + b$.

$$\begin{aligned} 45 &= 5(9)+b \\ 45 &= 45+b \\ b &= 45-45 \\ &= 0 \end{aligned}$$

The equation for the relation is $y = 5x$.

Look at the graph in Example 8 to see that the y-intercept really is -5.

Is the y-intercept really $+1$? Look at the graph in Example 9 to see that it is.

Now, in the following questions, you will try to draw graphs and write the equations of graphs. In each of the questions, a point and the slope of the graph of a linear relation is given. Graph paper is provided in **Appendix B**. Do at least the odd-numbered questions.

In the following three questions, draw the graph.

1. A point of the graph is $(-3, 1)$, and the slope is $\frac{1}{4}$.
2. A point of the graph is $(-1, -2)$, and the slope is -3.
3. A point of the graph is $(-2, 0)$, and the slope is $\frac{7}{8}$.

In the next five questions, write the equation for the relation.

4. A point of the graph is $(-1, 4)$, and the slope is 1.
5. A point of the graph is $(-1, -2)$, and the slope is -3.
6. A point of the graph is $(2, 7)$, and the slope is 1.
7. A point of the graph is $(3, -2)$, and the slope is -2.
8. A point of the graph is $(11, 9)$, and the slope is $\frac{2}{3}$.

In the last two questions, write the equation and draw the graph.

9. A point of the graph is $(-4, -4)$, and the slope is 2.
10. A point of the graph is $(-8, 13)$, and the slope is $\frac{-3}{4}$.

For solutions to **Activity 2**, turn to **Appendix A**, **Topic 1**.

Activity 3

Write the equation and draw the graph of a linear relation given two points that are on the graph of the relation.

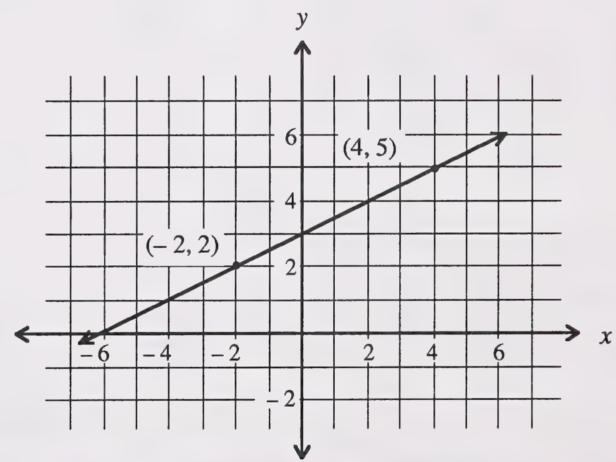
You can draw the graph of a linear relation when you are given two points on the graph. You plot the points and then you draw a line through them.

This concept is shown in the following examples.

Example 13

The points (4, 5) and (– 2, 2) are on the graph of a linear relation. Draw the graph.

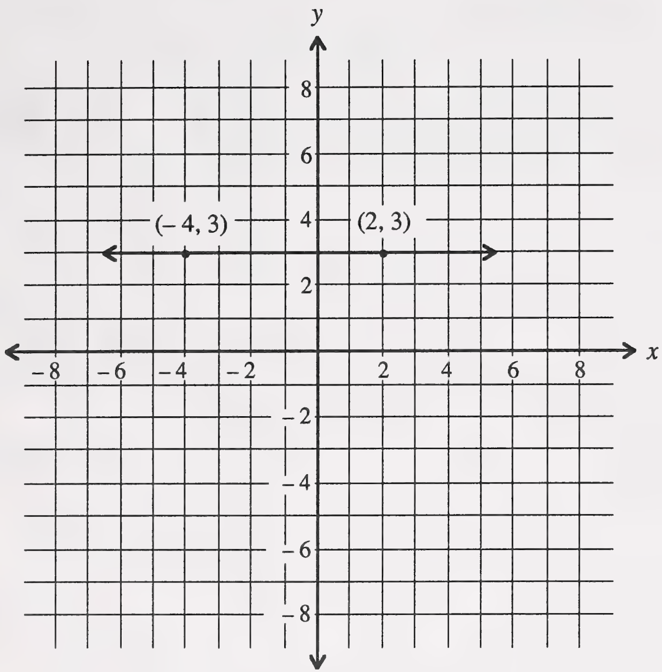
Solution:



Example 14

The points (– 4, 3) and (2, 3) are on the graph of a linear relation. Draw the graph.

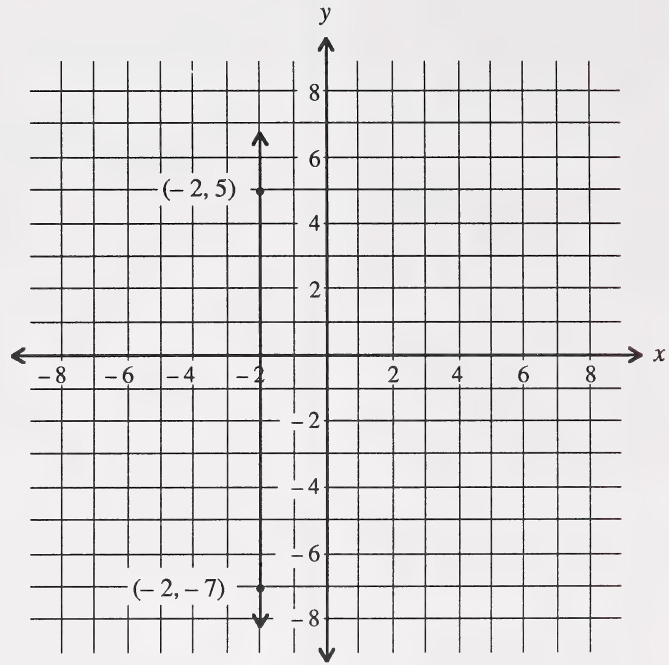
Solution:



Example 15

Draw the graph of the linear relation which has the points (– 2, – 7) and (– 2, 5) on it.

Solution:



When you are given two points on the graph of a linear relation, you can write the equation of the relation. When the points have the same x-coordinates or the same y-coordinates, the equation is obtained in a straightforward manner. This is shown in the next two examples.

For the graph, go back to Example 14. Since two points on the line have the same x-coordinates, so must all the other points.

Example 16

Write the equation of the linear relation whose graph includes the points $(-2, -7)$ and $(-2, 5)$.

Solution:

The x-coordinates are the same for these points.
The equation is $x = -2$.

The equation $x = -2$ cannot be put into the $y = mx + b$ form.

For the graph, go back to Example 13. Since two points on the line have the same y-coordinates, so must all the other points.

Example 17

The points $(-4, 3)$ and $(2, 3)$ are on the graph of a linear equation.
Write the equation of the relation.

Solution:

The y-coordinates are the same for these points.
The equation is $y = 3$.

The equation $y = 3$ is in the $y = mx + b$ form. Here, $m = 0$ and $b = 3$.

You can also write the equation for the linear equation when the given points have no coordinates in common. In that case, you first determine the slope of the graph of the relation using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$ where (x_1, y_1) and (x_2, y_2) are the two points on the graph of the relation.

You can think of the slope as the "rise" over the "run."

Once you have the slope, the problem becomes one of writing the equation given one point and the slope. You have done this in **Activity 2**. If one of the given points is a y-intercept, you can use the method used in **Activity 1**.

See how all this is applied in the next examples.

Example 18

Write the equation of the linear relation whose graph includes the points (2, 1) and (8, 4).

Solution:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-1}{8-2} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

You now look for the equation of the line running through (2, 1) and having slope $\frac{1}{2}$.

$$\begin{array}{c} \tfrac{1}{2} \\ \downarrow \end{array}$$

$$y = mx + b$$

$$\begin{array}{ccc} \uparrow & & \uparrow \\ 1 & & 2 \end{array}$$

$$\begin{aligned} 1 &= \frac{1}{2}(2) + b \\ 1 &= 1 + b \\ b &= 0 \end{aligned}$$

The equation is $y = \frac{1}{2}x + 0$ or $y = \frac{1}{2}x$.

Example 19

A linear relation has a graph which includes the points (0, 4) and (8, – 2).
Write the equation for the relation.

Solution:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-(-2)}{0-8} \\ &= \frac{6}{-8} \\ &= \frac{-3}{4} \end{aligned}$$

The point (0, 4) tells you that the y-intercept is 4.

The equation is $y = \frac{-3}{4}x + 4$.

In Example 19, (8, – 2) is considered the first ordered pair and (0, 4) the second ordered pair. You can designate either ordered pair as your first ordered pair.

You can also look for the equation of the line running through (8, 4) and having slope $\frac{1}{2}$. Try it. You will get the same equation.

Example 20

Suppose the graph of a relation is a line through the points (3, 4) and (1, – 4). Write the equation for the relation.

Solution:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4-4}{1-3} \\ &= \frac{-8}{-2} \\ &= 4 \end{aligned}$$

$$\underset{\substack{\uparrow \\ 4}}{y} = \overset{\substack{4 \\ \downarrow}}{m}\underset{\substack{\uparrow \\ 3}}{x} + b$$

$$\begin{aligned} 4 &= 4(3) + b \\ 4 &= 12 + b \\ b &= -8 \end{aligned}$$

The equation is $y = 4x - 8$.

Now try to draw the graphs and write the equations in the following questions. In each question, two points of the graph of a linear equation are given. Graph paper is provided in **Appendix B**. Do the odd-numbered questions.

In the following two questions, draw the graphs of the relations.

1. Two points of the graph are (0, 0) and (8, 5).
2. Two points of the graph are (– 5, – 6) and (– 2, – 4).

In the next five questions, write the equations of the relations.

3. Two points of the graph are (0, 0) and (8, 5).
4. Two points of the graph are (16, 3) and (32, – 4).
5. Two points of the graph are (–1, –1) and (7, 5).
6. Two points of the graph are (3, – 4) and (27, – 4).
7. Two points of the graph are (8, 6) and (8, 1).

In each of the next three questions, write the equation and draw the graph of the relation.

8. Two points of the graph are (5, – 8) and (5, 7).
9. Two points of the graph are (– 5, – 6) and (–1, 0).
10. Two points of the graph are (3, – 5) and (–1, 3).

For solutions to **Activity 3**, turn to **Appendix A, Topic 1**.

Activity 4

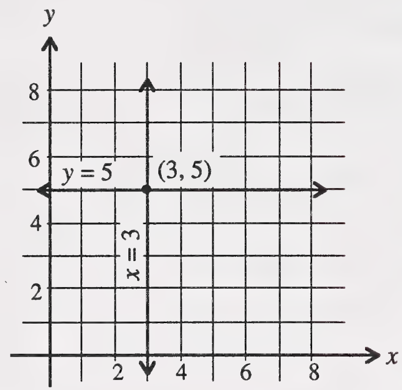
Write the equation and draw the graph of a linear relation given one point and the equation of a line that is parallel or perpendicular to the required linear relation.

By geometric construction, you know that you can determine a line through a point parallel or perpendicular to a given line. It should come as no surprise that you can do the same algebraically.

Before going further, it would be helpful to agree on some terminology. A vertical line is parallel to the y-axis and a horizontal line is parallel to the x-axis. Equations or relations can be said to be parallel or perpendicular when their graphs have such an orientation.

Suppose you have to write an equation of a vertical or horizontal line through point (x_1, y_1). The equation of the vertical line will be $x = x_1$ and the equation of the horizontal line will be $y = y_1$.

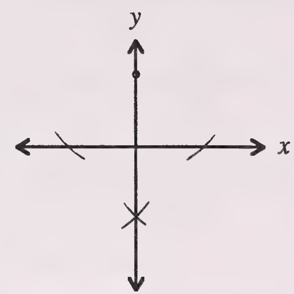
For example, the vertical line passing through (3, 5) is $x = 3$ and the horizontal line passing through (3, 5) is $y = 5$.



If you were given a vertical line, then another line parallel to it would also be vertical, whereas, another line perpendicular to it would be horizontal.

If the given line were horizontal, then another line parallel to it would also be horizontal and another line perpendicular to it would be vertical. You can see how this is applied in the next examples.

Construction of a line through a point **parallel** to a given line.



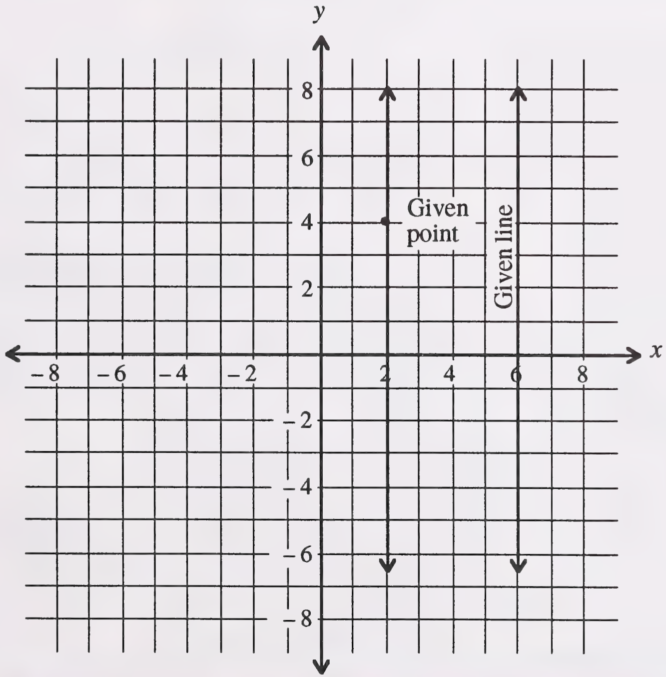
Construction of a line through a point **perpendicular** to a given line.

Example 21

Draw the graph and write the equation of a vertical line passing through (2, 4) and parallel to $x = 6$.

Solution:

The graph of $x = 6$ is a vertical line. The graph parallel to $x = 6$ is also a vertical line. So, the line passing through (2, 4) and parallel to $x = 6$ is a vertical line passing through (2, 4) as shown. You can recognize the equation of this line as $x = 2$.

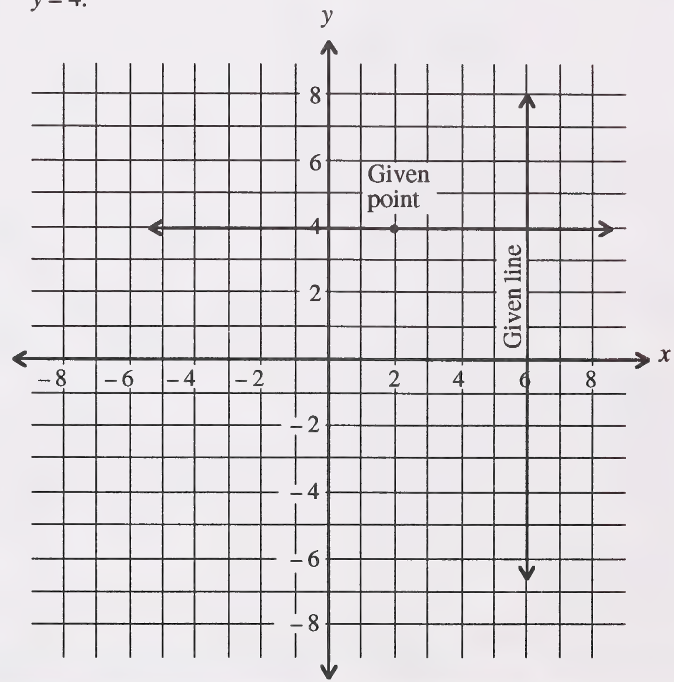


Example 22

Draw the graph and write the equation of a line passing through (2, 4) and perpendicular to $x = 6$.

Solution:

Since $x = 6$ is a vertical line, the graph perpendicular to it will be horizontal. If it follows the line passing through (2, 4) and is perpendicular to $x = 6$, it is a horizontal line passing through (2, 4) as shown. The equation of this line can be seen to be $y = 4$.

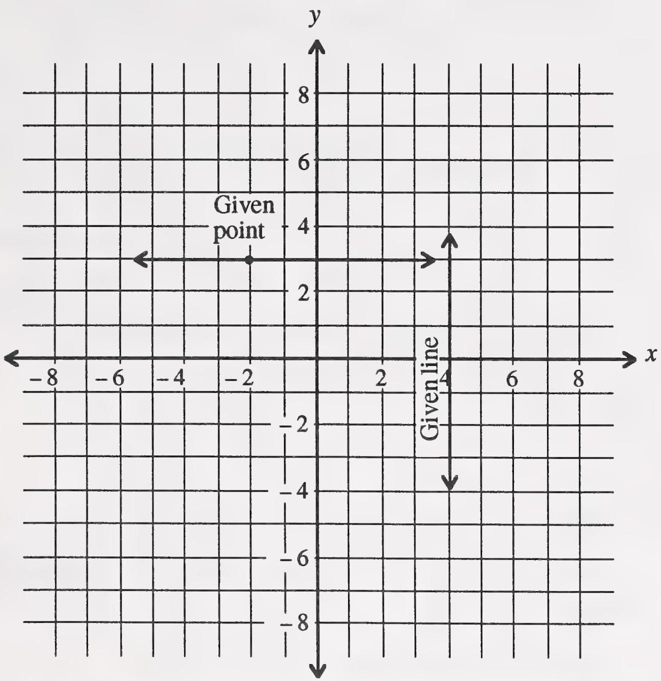


Example 23

Draw the graph and write the equation of a line passing through (– 2, 3) and perpendicular to $x = 4$.

Solution:

The given relation is vertical, so the line passing through (– 2, 3) and perpendicular to $x = 4$ is a horizontal line passing through (– 2, 3). The equation of such a line is $y = 3$.



Now try the following questions. Do the odd-numbered questions. If you want more practice, do the even-numbered questions as well. Graph paper is provided in **Appendix B**.

1. Write the equation and draw the graph of a line passing through (5, 4) and perpendicular to $x = 12$.

2. Write the equation and draw the graph of a line passing through (– 5, – 2) and perpendicular to $y = – 2$.

3. Write the equation and draw the graph of a line passing through (3, 4) and parallel to $x = – 3$.

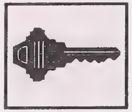
4. Write the equation and draw the graph of a line passing through (–1, – 8) and parallel to $y = 6$.

5. Write the equation and draw the graph of a line passing through (5, 7) and perpendicular to $y = 9$.

6. Write the equation of a linear relation which includes (–18, 20) and is perpendicular to the relation $y = 100$.

7. Write the equation of a linear relation which includes (34, – 273) and is parallel to $x = 15$.

For solutions to **Activity 4**, turn to **Appendix A**, **Topic 1**.



So far in this topic, you have determined graphs and equations for linear relations when only vertical or horizontal lines were involved. Next, you will include situations in which the given lines are oblique to the x- and y-axis. Then you use the fact that parallel lines have the same slope and for perpendicular lines, the product of the slopes is –1. For **parallel lines** $m_1 = m_2$ and for **perpendicular lines** $m_1 \times m_2 = -1$. (You could not use these slope relations in dealing with the perpendicular of a horizontal line or a line parallel to a vertical line because the slope would not be defined.)

You can see how these slope relations are applied in the next examples.

Example 24

Write the equation and draw the graph of a line passing through (4, –1) and perpendicular to $y = 2x + 3$.

Solution:

For the perpendicular line, the slope is

$$\begin{aligned} m_2 &= \frac{-1}{m_1} \\ &= -\frac{1}{2}. \end{aligned}$$

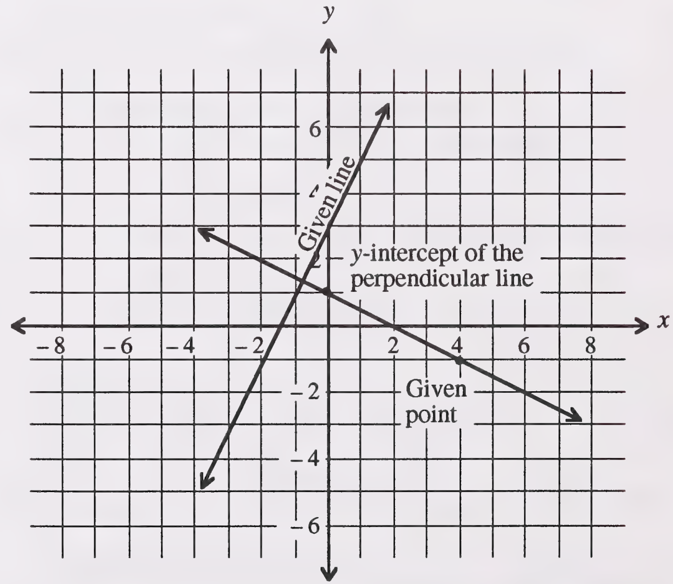
$$\underset{\substack{\uparrow \\ -1}}{y} = \overset{\substack{-\frac{1}{2} \\ \downarrow}}{m}\underset{\substack{\uparrow \\ 4}}{x} + b$$

$$-1 = -\frac{1}{2}(4) + b$$

$$-1 = -2 + b$$

$$b = 1$$

The equation of the line is $y = -\frac{1}{2}x + 1$.
Since $b = 1$, the y-intercept is (0, 1).



To graph the perpendicular line, draw a line through the given point and the y-intercept. (Note: Use your protractor to verify that these lines are perpendicular.

Example 25

Write the equation and draw the graph of a line passing through $(-3, -4)$ and parallel to $y = \frac{1}{3}x + 2$.

Solution:

For the parallel line, the slope is $\frac{1}{3}$.

$$\underset{\substack{\uparrow \\ -4}}{y} = \overset{\substack{\frac{1}{3} \\ \downarrow}}{m}\underset{\substack{\uparrow \\ -3}}{x} + b$$

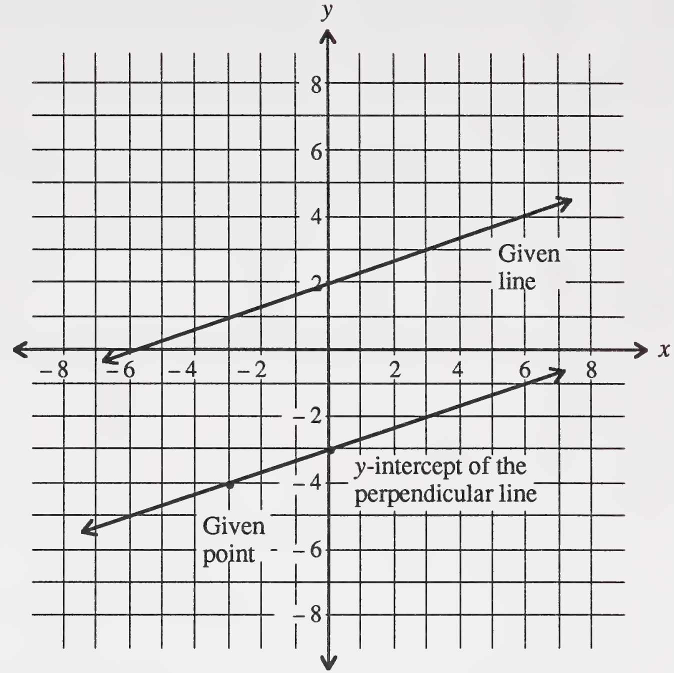
$$-4 = \frac{1}{3}(-3) + b$$

$$-4 = -1 + b$$

$$b = -3$$

The equation of the line is $y = \frac{1}{3}x - 3$.

Since $b = -3$, the y-intercept is $(0, -3)$. To graph the parallel line, draw a line through the given point and the y-intercept.



It may occur that the given point is the y-intercept of the line to be graphed. In that case, the slope and y-intercept method of the first activity in this topic can be used. This is demonstrated in the next example.

Example 26

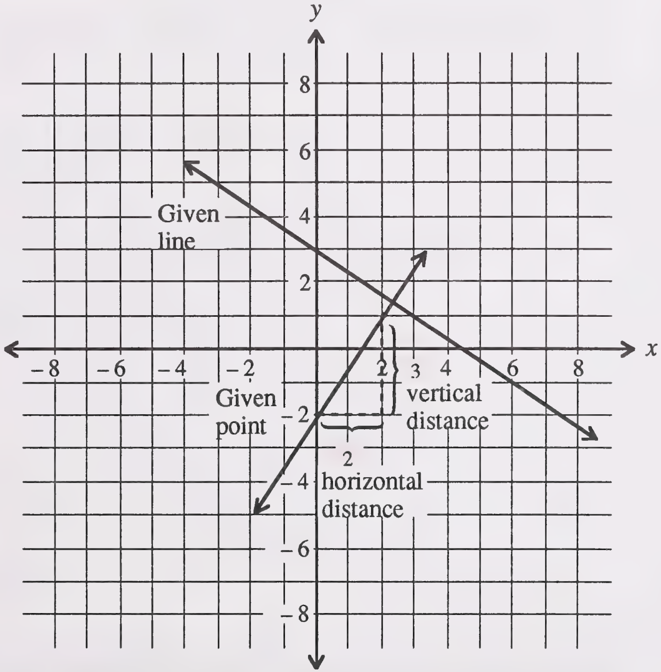
Write the equation and draw the graph of a line passing through $(0, -2)$ and perpendicular to $y = -\frac{2}{3}x + 3$.

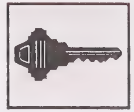
Solution:

For the perpendicular line, the slope is $m_2 = \frac{-1}{m_1} = \frac{-1}{\left(-\frac{2}{3}\right)} = \frac{3}{2}$.

Since $(0, -2)$ is on the line, -2 is the y-intercept.

The equation of the line is $y = \frac{3}{2}x - 2$.





The equation of the given line may be in the form $Ax + By = -C$. For oblique lines, A and B will be non-zero and the equation can be converted into the form $y = mx + b$ by solving for y. Then, you can proceed as before.

Look at the following example.

Example 27

Write the equation of a line passing through $(0, 5)$ and perpendicular to $3x - 4y = 28$.

Solution:

The given equation is $3x - 4y = -28$.

$$\begin{aligned} -4y &= -3x - 28 \\ 4y &= 3x + 28 \\ y &= \frac{3}{4}x + 7 \end{aligned}$$

The perpendicular line has a slope of

$$\begin{aligned} m_2 &= \frac{-1}{m_1} \\ &= \frac{-1}{\frac{3}{4}} \\ &= \frac{-4}{3}. \end{aligned}$$

Since $(0, 5)$ is on the line, the y-intercept is 5. The equation of the line is $y = -\frac{4}{3}x + 5$.

Now try the following questions. Do the odd-numbered questions. want more practice, do the remaining questions. Graph paper is provided in **Appendix B**.

8. Write the equation and draw the graph of a line passing through (0, 0) and parallel to $y = \frac{1}{8}x + 7$.

9. Write the equation and draw the graph of a line passing through (5, – 5) and parallel to $y = \frac{1}{5}x$.

10. Write the equation and draw the graph of a line passing through (4, – 1) and perpendicular to $y = -4x + 3$.

11. Write the equation and draw the graph of the line passing through (10, 3) and perpendicular to $y = \frac{-5}{3}x + 9$.

12. Write the equation and draw the graph of a line passing through (0, – 4) and perpendicular to $y = \frac{1}{2}x + \frac{3}{2}$.

13. Write the equation and draw the graph of a line passing through (– 6, – 3) and parallel to $3x - y = -11$.

14. Write the equation of a line passing through (1, 2) and perpendicular to $3x + 4y = 10$.

15. Write the equation of a line passing through (7, 4) and parallel to $y = 10$.

16. Write the equation of a line passing through (– 5, 3) and perpendicular to $y = -\frac{3}{2}$.

For solutions to **Activity 4**, turn to **Appendix A, Topic 1**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

You may decide to do both.

Extra Help

In this topic, you found that the equation of a line could be written when you were given certain conditions. This is reviewed below using some different notation.

1. Suppose the given slope is m_0 and the y-intercept is b_0. Then the equation of the line is $y = m_0 x + b_0$.

Example 28

Write the equation for the linear relation having a graph with a slope of $\frac{2}{7}$ and a y-intercept of 9.

Solution:

$m_0 = \frac{2}{7}$ and $b_0 = 9$ in $y = m_0 x + b_0$

The equation is $y = \frac{2}{7}x + 9$.

2. The given slope is m_0 and a point on the line is given as (x_0, y_0). The equation of the line is $y_0 = m_0 x_0 + b_0$, where $b_0 = y_0 - m_0 x_0$.

Letters with subscripts represent specific values.

Example 29

Write the equation for the linear relation having a graph with a slope of $\frac{3}{7}$ and containing the point (14, 4).

Solution:

$(x_0, y_0) = (14, 4)$, so $x_0 = 14$ and $y_0 = 4$.

$$\begin{aligned} b_0 &= y_0 - m_0 x_0 \\ &= 4 - \left(\tfrac{3}{7}\right)14 \\ &= 4 - 6 \\ &= -2 \end{aligned}$$

So, $y = m_0 x + b_0, m_0 = \frac{3}{7}$, and $b_0 = -2$.

The equation is $y = \frac{3}{7}x - 2$.

3. Suppose one point on the line is (x_1, y_1) and the other is (x_2, y_2). If $x_1 = x_2$ where $y_1 \neq y_2$, then the equation of the line is $x = x_1$. Otherwise, $x_1 \neq x_2$ and $y_1 \neq y_2$. The equation of the line is $y = m_0 x + b_0$, where

   $m_0 = \dfrac{y_2 - y_1}{x_2 - x_1}$ and $b_0 = y_1 - m_0 x_1$.

   In this case, the points have the same x-coordinate and the line through the points would not have a defined slope.

   Either point can be used to determine b_0. So $b_0 = y_2 - m_0 x_2$.

Example 30

Write the equation of a linear equation where the graph includes the points (– 4, – 5) and (5, 13).

Solution:

$$\begin{aligned} m_0 &= \frac{13-(-5)}{5-(-4)} \\ &= \frac{18}{9} \\ &= 2 \end{aligned}$$

$$\begin{aligned} b_0 &= y_2 - m_0 x_2 \\ &= 13-2(5) \\ &= 13-10 \\ &= 3 \end{aligned}$$

So, in $y = m_0 + b_0$, $m_0 = 2$, and $b_0 = 3$.

The equation is $y = 2x + 3$.

If one point on the line is (x_0, y_0) and it is parallel to

$y = m_1 x + b_1$, then the equation of the line is $y = m_0 x + b_0$,

where $m_0 = m_1$ and $b_0 = y_0 - m_0 x_0$.

Example 31

Write the equation of a line that includes (2, 8) and is parallel to $y = 3x + 20$.

Solution:

$$\begin{aligned} m_0 &= 3 \\ b_0 &= y_0 - m_0 x_0 \\ &= 8-3(2) \\ &= 8-6 \\ &= 2 \end{aligned}$$

So, $y = m_0 x + b_0, m_0 = 3$, and $b_0 = 2$.

The equation is $y = 3x + 2$.

If one point on the line is (x_0, y_0) and it is parallel to

$y = m_1 x + b_1$, then the equation of the line is $y = m_0 x + b_0$,

where $m_0 = m_1$ and $b_0 = y_0 - m_0 x_0$.

Example 32

Write the equation of a line that includes (6, 10) and is perpendicular to $y = \frac{-3}{4}x + 25$.

Solution:

$$\begin{aligned} m_0 &= \frac{-1}{m_1} \\ &= -\frac{1}{\left(\frac{-3}{4}\right)} \\ &= \frac{4}{3} \end{aligned}$$

$$\begin{aligned} b_0 &= y_0 - m_0 x_0 \\ &= 10 - \frac{4}{3}(6) \\ &= 10 - 8 \\ &= 2 \end{aligned}$$

So, $y = m_0 x + b_0$, $m_0 = \frac{4}{3}$, and $b_0 = 2$.

The equation is $y = \frac{4}{3}x + 2$.

If one point of the line is (x_0, y_0) and it is parallel to $x = k$, then the equation of the line is $x = x_0$.

Example 33

Write the equation of the linear relation whose graph includes the point (– 4, 12) and which is parallel to $x = 50$.

Solution:

In $x = x_0$, $x_0 = -4$.
The equation is $x = -4$.

If one point of the line is (x_0, y_0) and it is perpendicular to $x = k$, then the equation of the line is $y = y_0$.

Example 34

Write the equation of the linear relation whose graph includes the point (–1, 12) and is perpendicular to $x = -23$.

Solution:

In $y = y_0$, $y_0 = 12$.
The equation of the line is $y = 12$.

If one point of the line is (x_0, y_0) and it is perpendicular to $y = k$, then the equation of the line is $x = x_0$.

In $x = k$, k is a constant. Examples of such equations are $x = -1$, $x = 4$, and $x = 10$. These cannot be presented in the form $y = mx + b$ because the slope is undefined.

Example 35

Write the equation of the linear relation whose graph includes the point (2, 5) and is perpendicular to $y = 1$.

Solution:

In $x = x_0$, $x_0 = 2$.
The equation of the line is $x = 2$.

Try the following questions.

1. Write the equation for the linear relation having a graph with a slope of 3 and a y-intercept of -2.
2. Write the equation for the linear relation having a slope of 2 and contains the point (14, 22).
3. Write the equation for the linear relation whose graph includes the points (2, – 1) and (2, 5).
4. Write the equation of a linear relation whose graph includes the points (2, 4) and (4, 10).
5. Write the equation of a linear relation whose graph includes the point (5, 14) and is parallel to $2x - y = -1$.
6. Write the equation of a linear relation whose graph includes the point (– 6, 7) and is perpendicular to $2x - y = -1$.
7. Write the equation of a linear relation whose graph includes the point (2, 3) and is parallel to $x = 0$.
8. Write the equation of a linear relation whose graph includes the point (2, 3) and is perpendicular to $y = -\frac{2}{3}$.
9. Write the equation of a linear relation whose graph includes the point (– 1, 7) and is perpendicular to $x = 13$.

For solutions to **Extra Help**, turn to **Appendix A**, **Topic 1**.

Hint:
Put the equation into the $y = mx + b$ form.

Extensions

The lowest possible temperature is called absolute zero. It was first determined by studying the volume of a sample of air at various temperatures. The relationship involving the volume and the temperature was found to be linear over a range of temperatures accessible to laboratories.

By assuming that the volume-temperature relationship remains linear, it is possible to calculate the temperature at which the volume would be zero. That temperature is absolute zero.

Find the value of absolute zero in **degrees Celsius** by doing the following questions.

1. A sample of air has a volume of 100 mL at 0° C. At 200° C, the volume of this same sample is 173 mL. With volume along the y-axis and temperature along the x-axis, plot two points to represent the data.

Remember to choose an appropriate scale.

2. Using the two points, graph the linear relation. (Graph paper is supplied on the next page.)
3. Write the equation of the linear relation using the two points.

This is the same as finding the x-intercept.

4. Calculate absolute zero from your equation.
5. Look at the value of absolute zero in a reference book. How close is your calculated value from the accepted value?

For solutions to **Extensions**, turn to **Appendix A, Topic 1**.

Unit Summary

What You Have Learned

In this unit, you have learned how to draw the graph and write the equation of a linear relation given certain "minimum" conditions that uniquely define it. These conditions could be any one of the following.

- the slope and the y-intercept
- any two points
- one point and the slope
- one point and the equation of a parallel or perpendicular line

You are now ready to complete the **Unit Assignment**.

Appendices

Appendix A
Solutions

Review

Topic 1 Equations and Graphs of Linear Relations

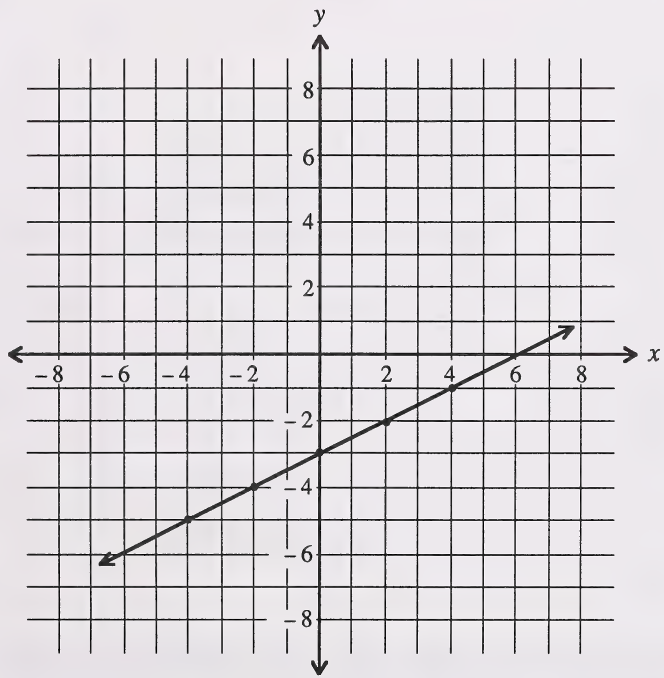
Appendix B

Graph Paper

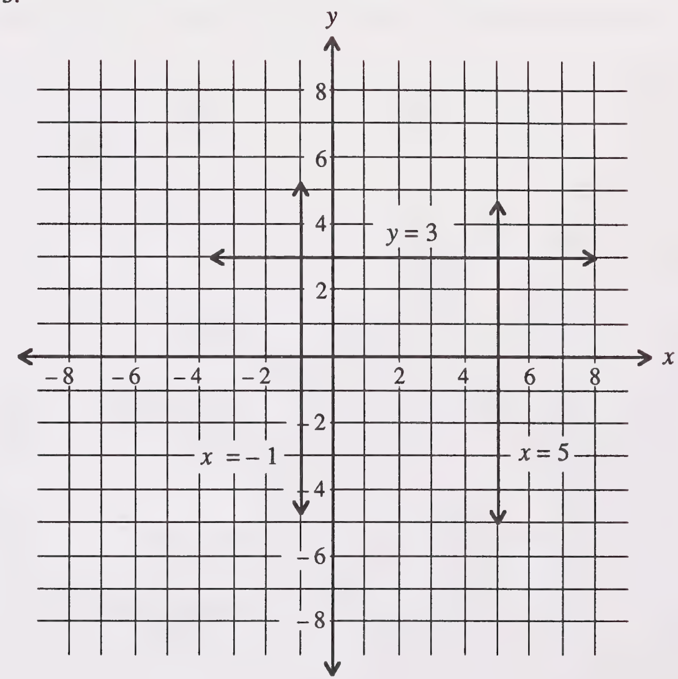
Review

1. c. The variables x and y are not raised to a power greater than the first power.

2. One set of ordered pairs is $\{(-4, -5), (-2, -4), (0, -3), (2, -2), (4, -1)\}$.



3.



4. x-intercept
Let $y = 0$ in $y = \frac{1}{2}x + 3$.

$$0 = \frac{1}{2}x + 3$$

$$\frac{1}{2}x = -3$$

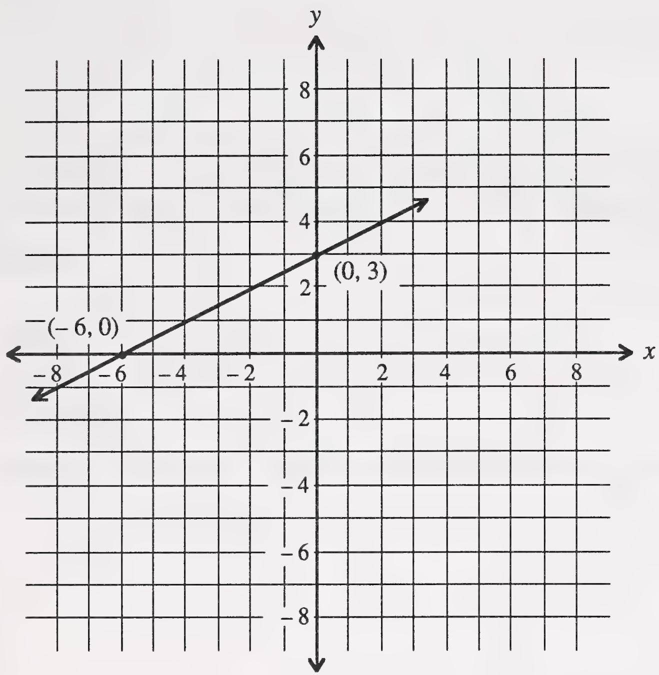
$$x = -6$$

y-intercept

Let $x = 0$ in $y = \frac{1}{2}x + 3$.

$$y = \frac{1}{2}(0) + 3$$

$$y = 3$$



5. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-15-3}{4-(-5)}$$

$$= \frac{-18}{9}$$

$$= -2$$

6. Put the given equation in the $y = mx + b$ form.

$$x - 3y = -3$$
$$-3y = -x - 3$$
$$3y = x + 3$$
$$y = \frac{1}{3}x + 1$$

The slope of any parallel line is $\frac{1}{3}$.

Put $4x - 12y = -3$ in the $y = mx + b$ form.

$$-12y = -4x - 3$$
$$12y = 4x + 3$$
$$y = \frac{4}{12}x + \frac{3}{12}$$
$$y = \frac{1}{3}x + \frac{1}{4}$$

The slope of this line is also $\frac{1}{3}$.

Therefore, equation b has a graph that is parallel to $x - 3y = -3$.

7. The slope of the given line is $\frac{3}{4}$.
The slope of the perpendicular to this line is

$$\begin{aligned} m_1 &= \frac{-1}{m_2} \\ &= -\frac{1}{\frac{3}{4}} \\ &= -\frac{4}{3}. \end{aligned}$$

This is the slope of the graph of $y = -\frac{4}{3}x + 4$.

Therefore, equation a has a graph that is perpendicular to $y = \frac{3}{4}x + 10$.

Exploring Topic 1

Activity 1

Write the equation and draw the graph of a linear relation given the slope and the y-intercept of the graph of the relation.

1. $y = 5x + 4$

2. $y = -\frac{2}{3}x - 49$

3. $y = 25x$

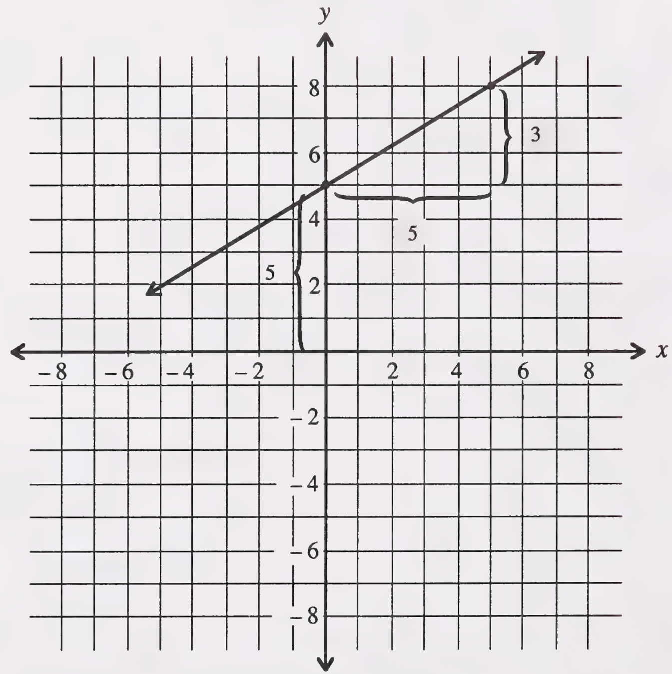
4. $y = 0x + 19$ or $y = 19$

5. $y = -4x + 3$

6. $y = \frac{1}{5}x - \frac{1}{10}$

$$\begin{aligned} -\frac{1}{5}x + y &= -\frac{1}{10} \\ -2x + 10y &= -1 \\ 2x - 10y &= 1 \end{aligned}$$

7



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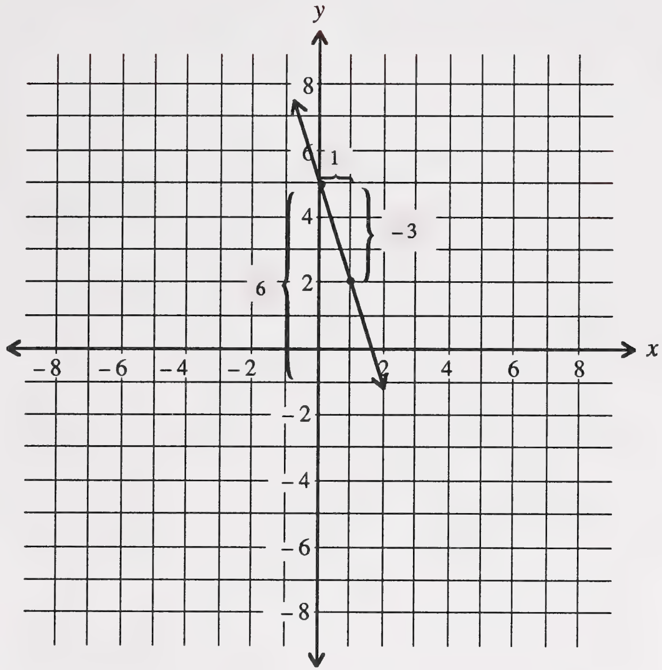

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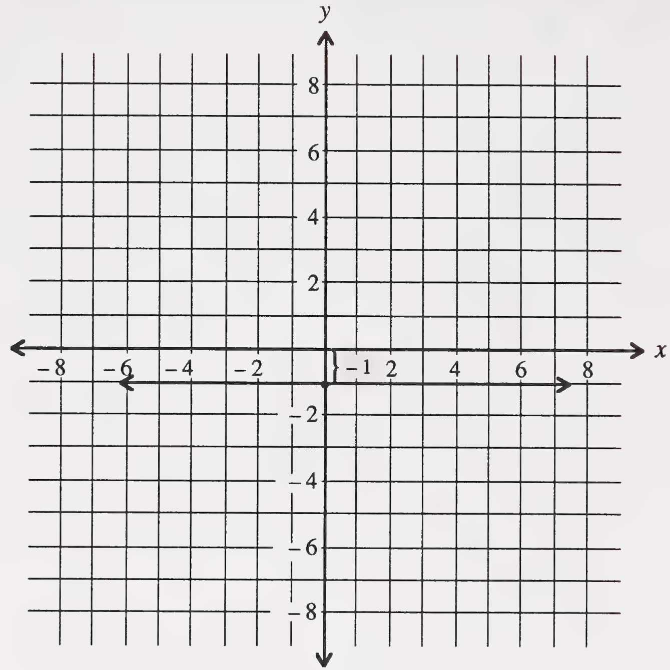

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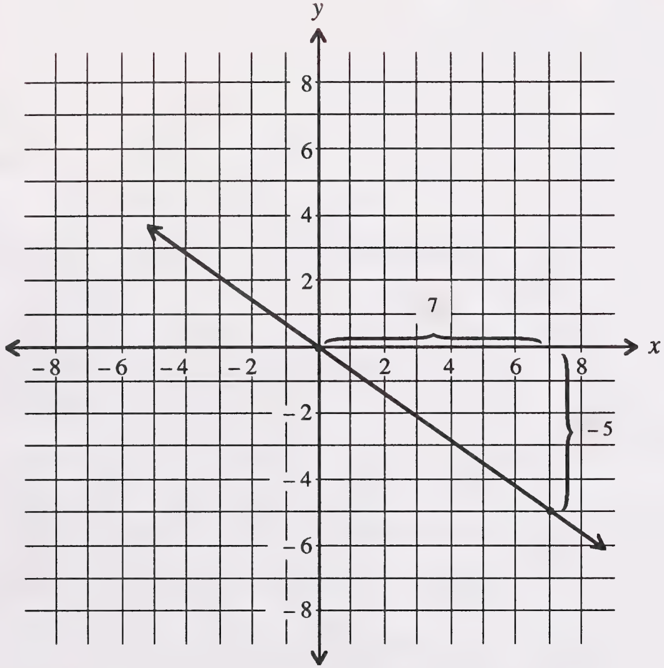
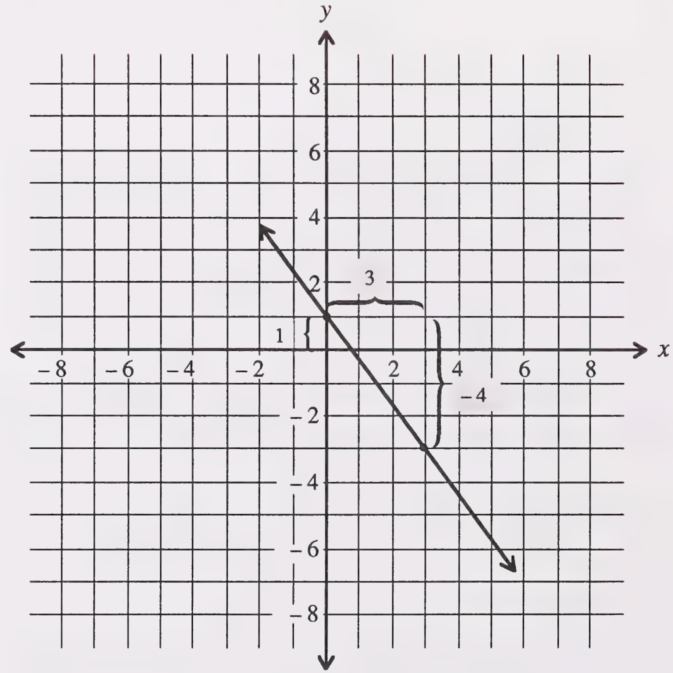


11. $y = -1\frac{1}{3}x + 1$

$m = -1\frac{1}{3}$

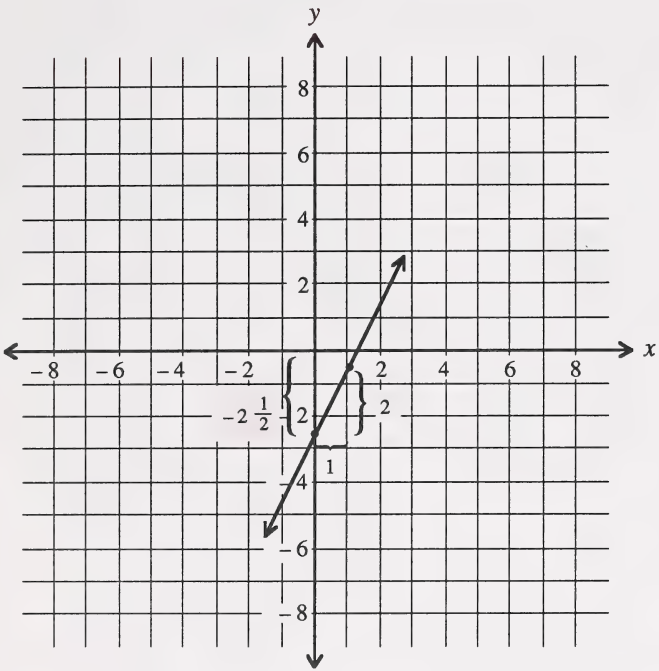
$= \frac{-4}{3}$



12. $y = 2x - 2\frac{1}{2}$

$m = 2$

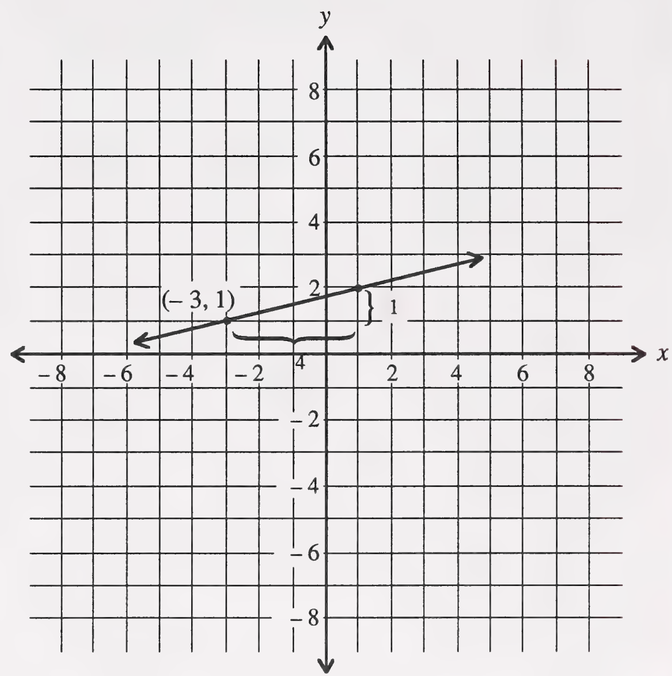
$= \frac{2}{1}.$



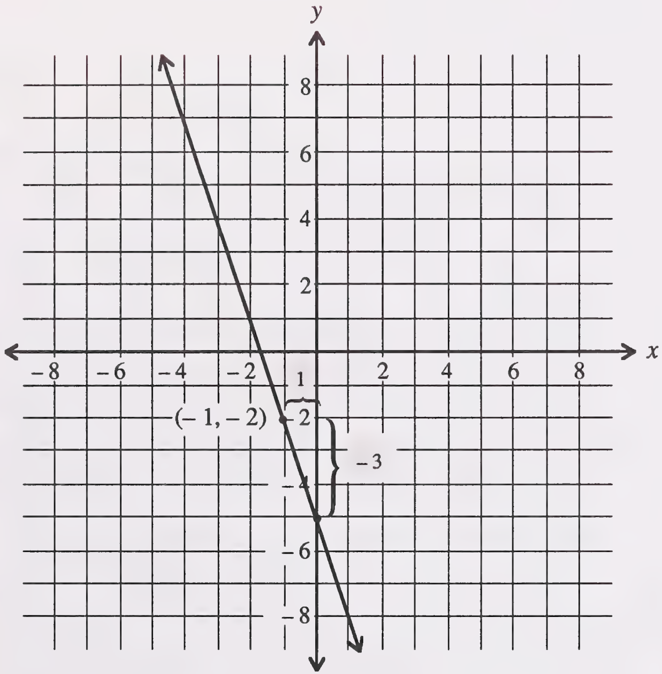
Activity 2

Write the equation and draw the graph of a linear relation given one point and the slope of the graph of the relation.

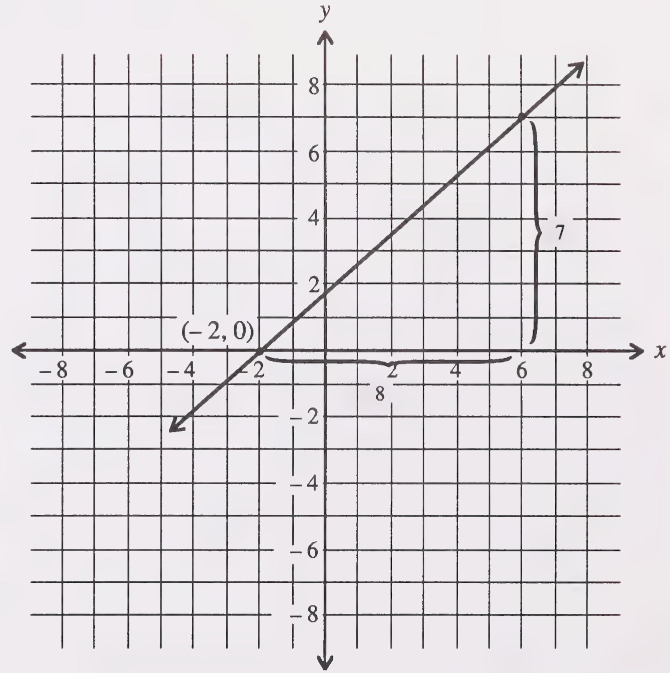
1.



2.



3.



4.

$$\boxed{\underset{\underset{4}{\uparrow}}{y} = \overset{\overset{1}{\downarrow}}{m}\underset{\underset{-1}{\uparrow}}{x} + b}$$

$4 = 1(-1) + b$
$4 = -1 + b$
$b = 5$

$y = 1x + 5$ or $y = x + 5$

5.

$$\boxed{\underset{\underset{-2}{\uparrow}}{y} = \overset{\overset{-3}{\downarrow}}{m}\underset{\underset{-1}{\uparrow}}{x} + b}$$

$-2 = (-3)(-1) + b$
$-2 = 3 + b$
$b = -5$

$y = -3x - 5$

6.

$$\boxed{\underset{\underset{7}{\uparrow}}{y} = \overset{\overset{1}{\downarrow}}{m}\underset{\underset{2}{\uparrow}}{x} + b}$$

$7 = 1(2) + b$
$7 = 2 + b$
$b = 5$

$y = x + 5$

7.

$$\boxed{\underset{\underset{-2}{\uparrow}}{y} = \overset{\overset{-2}{\downarrow}}{m}\underset{\underset{3}{\uparrow}}{x} + b}$$

$-2 = (-2)(3) + b$
$-2 = -6 + b$
$b = 4$

$y = -2x + 4$

8.

$$\boxed{\underset{\underset{9}{\uparrow}}{y} = \overset{\overset{\frac{2}{3}}{\downarrow}}{m}\underset{\underset{11}{\uparrow}}{x} + b}$$

$9 = \left(\frac{2}{3}\right)(11) + b$

$9 = \frac{22}{3} + b$

$9 = 7\frac{1}{3} + b$

$b = 9 - 7\frac{1}{3}$

$\quad = 1\frac{2}{3}$

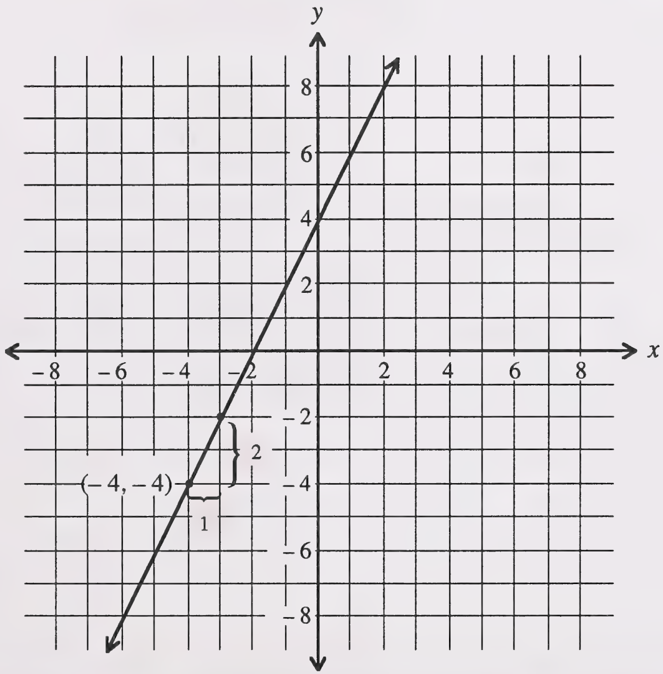
$y = \frac{2}{3}x + 1\frac{2}{3}$

9.

$$\boxed{\begin{array}{c} 2 \\ \downarrow \\ y = mx + b \\ \uparrow \quad\;\; \uparrow \\ -4 \quad -4 \end{array}}$$

$-4 = 2(-4) + b$
$-4 = -8 + b$
$b = 4$

$y = 2x + 4$



10.

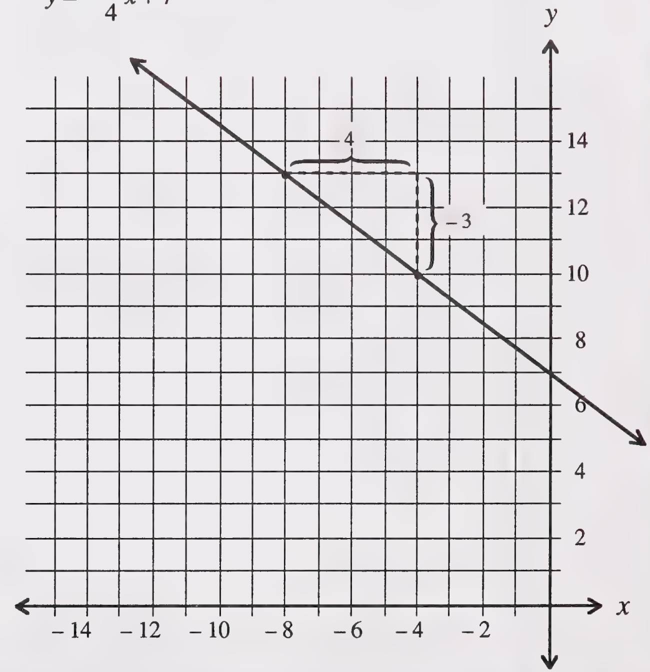
$$\boxed{\begin{array}{c} -\frac{3}{4} \\ \downarrow \\ y = mx + b \\ \uparrow \quad\;\; \uparrow \\ 13 \quad -8 \end{array}}$$

$13 = \left(-\frac{3}{4}\right)(-8) + b$

$13 = 6 + b$

$b = 7$

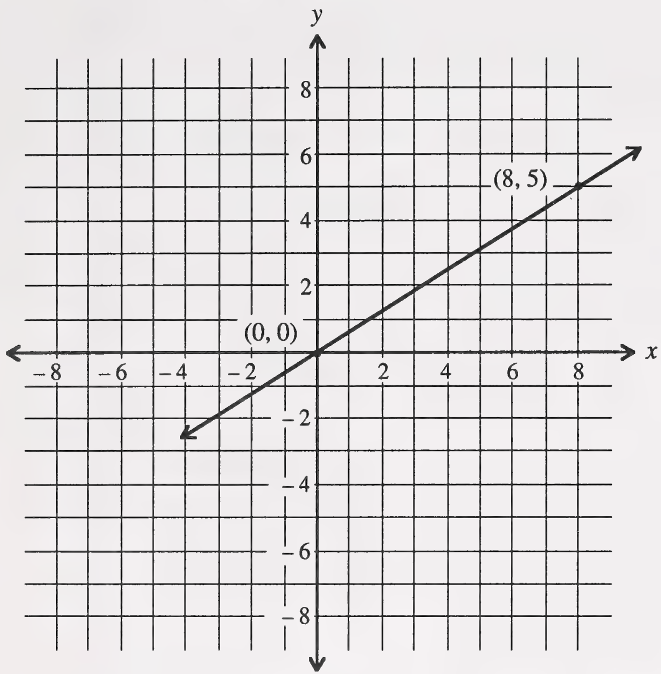
$y = -\frac{3}{4}x + 7$



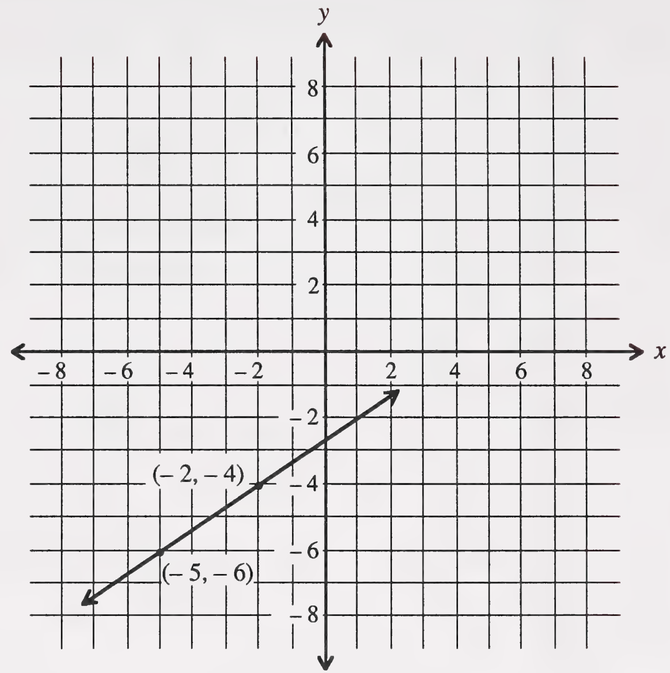
Activity 3

Write the equation and draw the graph of a linear relation given two points that are on the graph of the relation.

1.



2.



3. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{5-0}{8-0}$

$= \dfrac{5}{8}$

$$\boxed{\begin{array}{c} \frac{5}{8} \\ \downarrow \\ \underset{\substack{\uparrow \\ 0}}{y} = m\underset{\substack{\uparrow \\ 0}}{x} + b \end{array}}$$

$0 = \left(\dfrac{5}{8}\right)(0) + b$

$0 = 0 + b$

$b = 0$

$y = \dfrac{5}{8}x + 0$ or $y = \dfrac{5}{8}x$

4. $m = \dfrac{-4-3}{32-16}$

$= \dfrac{-7}{16}$

$$\boxed{\begin{array}{c} \frac{-7}{16} \\ \downarrow \\ \underset{\substack{\uparrow \\ 3}}{y} = m\underset{\substack{\uparrow \\ 16}}{x} + b \end{array}}$$

$3 = \left(\dfrac{-7}{16}\right)16 + b$

$3 = -7 + b$

$b = 10$

$y = \dfrac{-7}{16}x + 10$

5. $m = \dfrac{5-(-1)}{7-(-1)}$

$= \dfrac{6}{8}$

$= \dfrac{3}{4}$

$$\boxed{\begin{array}{c} \tfrac{3}{4} \\ \downarrow \\ y = mx + b \\ \uparrow \quad \uparrow \\ -1 \quad -1 \end{array}}$$

$-1 = \left(\dfrac{3}{4}\right)(-1) + b$

$-1 = -\dfrac{3}{4} + b$

$b = -\dfrac{1}{4}$

$y = \dfrac{3}{4}x - \dfrac{1}{4}$

6. $m = \dfrac{-4-(-4)}{27-3}$

$= \dfrac{0}{24}$

$= 0$

$$\boxed{\begin{array}{c} 0 \\ \downarrow \\ y = mx + b \\ \uparrow \quad \uparrow \\ -4 \quad 3 \end{array}}$$

$-4 = 0\,(3) + b$

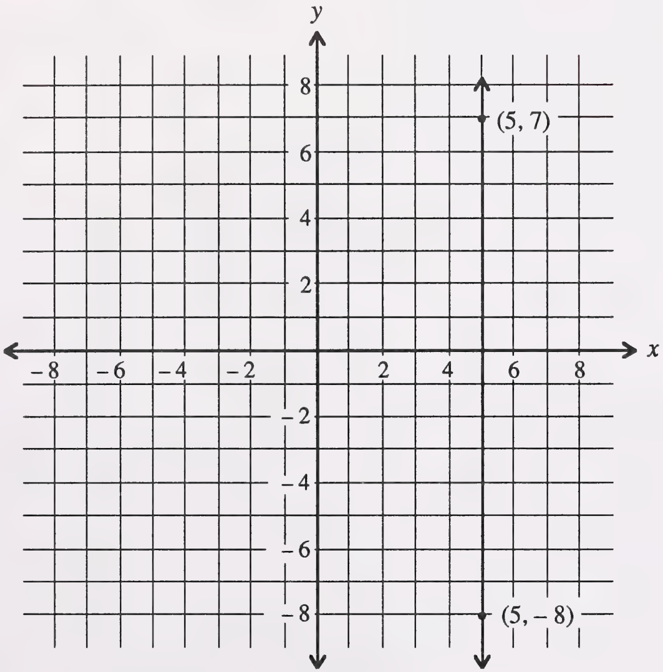
$-4 = 0 + b$

$b = -4$

$y = 0x - 4$ or $y = -4$

7. The x values of the given points are the same; therefore, the equation for the line is $x = 8$.

8. The given points have the same x-coordinate, so the equation of the line is $x = 5$.



9. $m = \dfrac{0-(-6)}{-1-(-5)}$

$= \dfrac{6}{4}$

$= \dfrac{3}{2}$

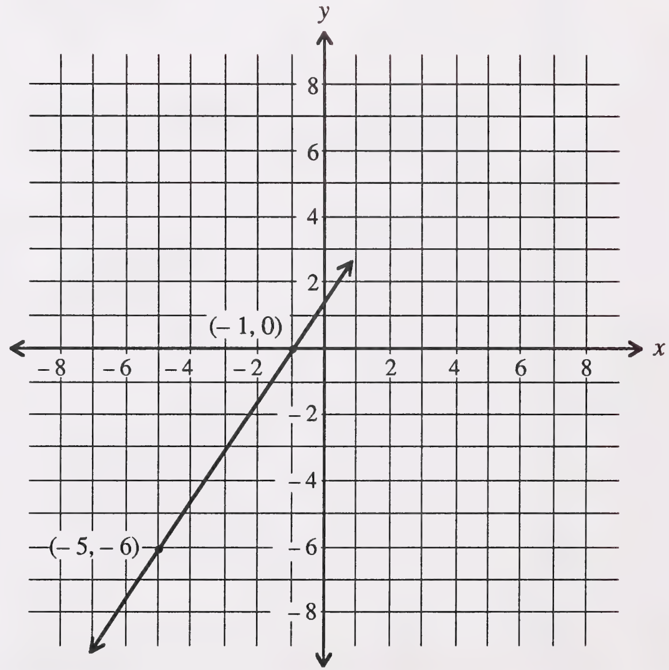
$$\begin{array}{c} \tfrac{3}{2} \\ \downarrow \\ y = mx + b \\ \uparrow \quad\quad \uparrow \\ 0 \quad\quad -1 \end{array}$$

$0 = \left(\dfrac{3}{2}\right)(-1) + b$

$0 = -\dfrac{3}{2} + b$

$b = \dfrac{3}{2}$

$y = \dfrac{3}{2}x + \dfrac{3}{2}$



10. $m = \frac{3-(-5)}{-1-3}$

$= \frac{8}{-4}$

$= -2$

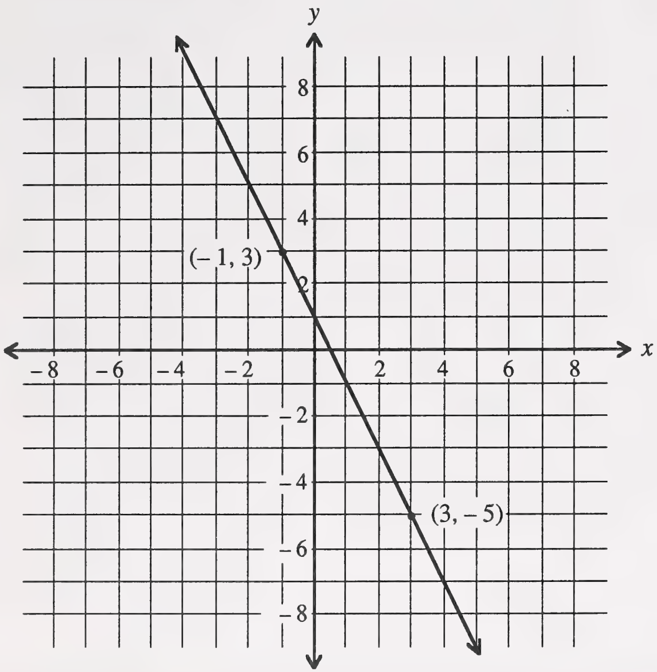
$$\begin{array}{c} -2 \\ \downarrow \\ y = mx + b \\ \uparrow \quad\quad \uparrow \\ 3 \quad\quad -1 \end{array}$$

$3 = (-2)(-1) + b$

$3 = 2 + b$

$b = 1$

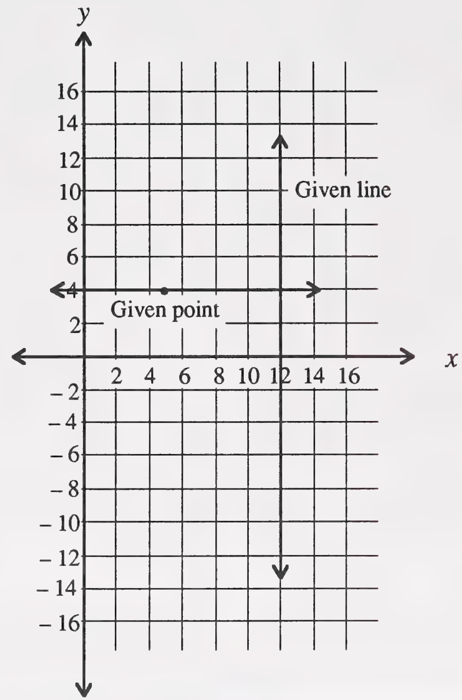
$y = -2x + 1$



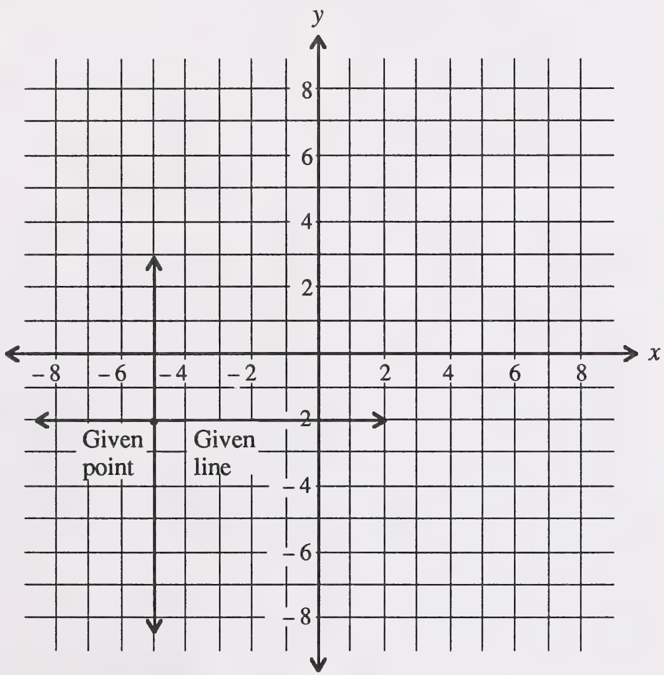
Activity 4

Write the equation and draw the graph of a linear relation given one point and the equation of a line that is parallel or perpendicular to the required relation.

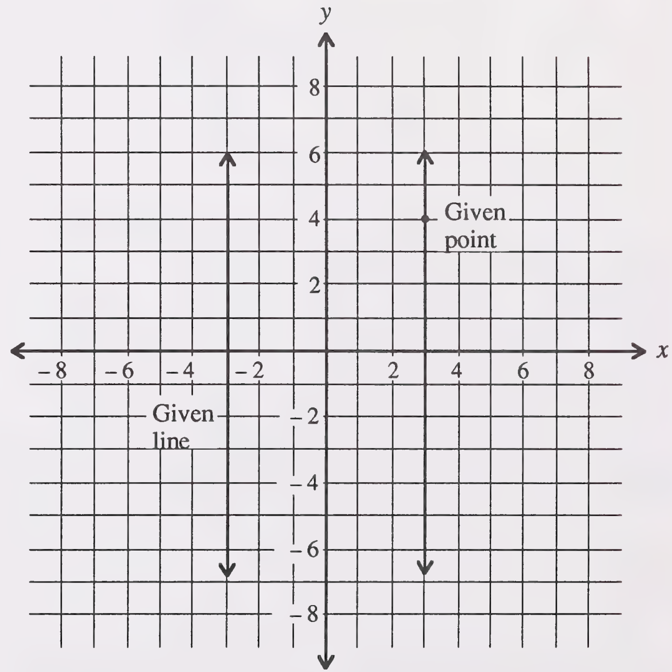
1. The given relation is vertical, so the line passing through (5, 4) and perpendicular to $x = 12$ is a horizontal line passing through (5, 4). The equation of such a line is $y = 4$.



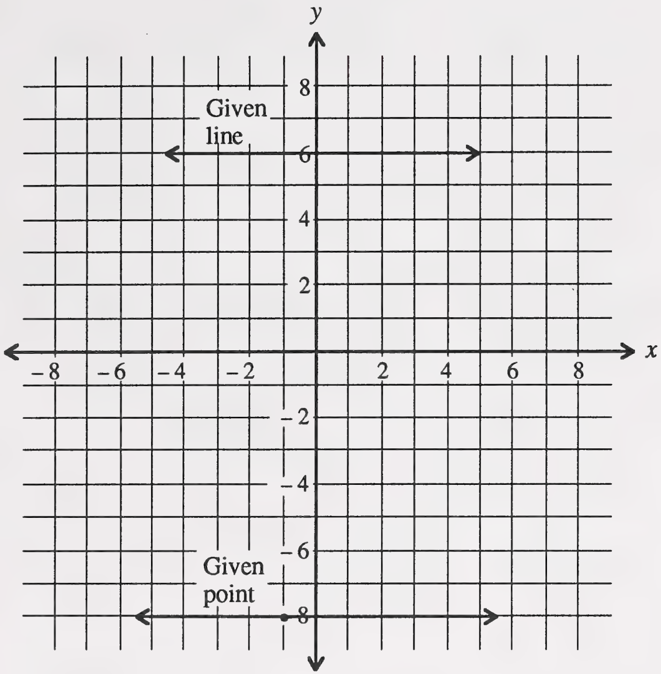
2. The given relation is horizontal, so the line passing through $(-5, -2)$ and perpendicular to $y = -2$ is a vertical line passing through $(-5, -2)$. The equation of such a line is $x = -5$.



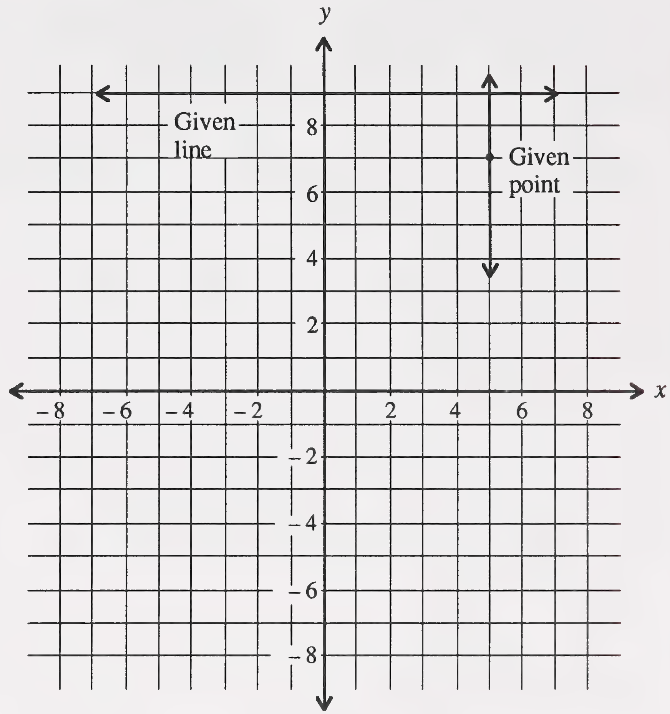
3. The given relation is vertical, so the line passing through $(3, 4)$ and parallel to $x = -3$ is a vertical line passing through $(3, 4)$. The equation of such a line is $x = 3$.



4. The given relation is horizontal, so the line passing through $(-1, -8)$ and parallel to $y = 6$ is a horizontal line passing through $(-1, -8)$. The equation of such a line is $y = -8$.



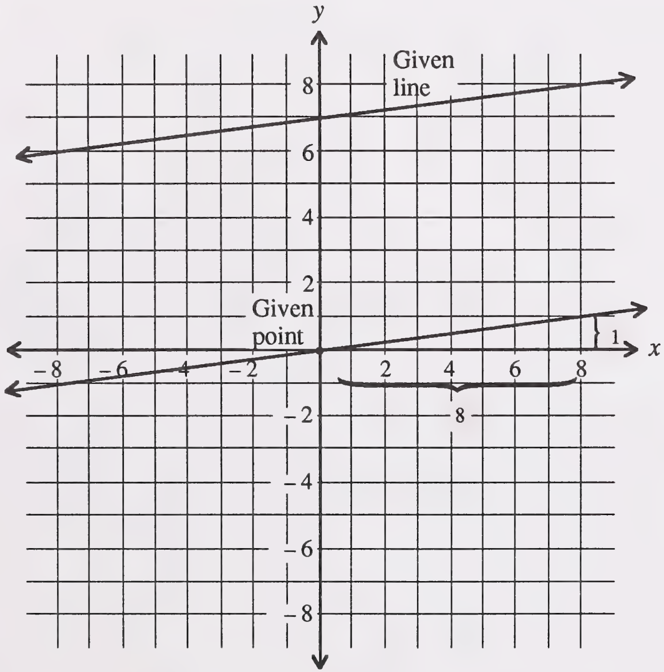
5. The given relation is horizontal, so the line passing through $(5, 7)$ and perpendicular to $y = 9$ is a vertical line passing through $(5, 7)$. The equation of such a line is $x = 5$.



6. The given relation is horizontal, so the line passing through $(-18, 20)$ and perpendicular to $y = 100$ is a vertical line running through $(-18, 20)$. The equation of such a line is $x = -18$.

7. The given relation is vertical, so the line passing through $(34, -273)$ and parallel to $x = 15$ is a vertical line passing through $(34, -273)$. The equation of such a line is $x = 34$.

8. For the parallel line, the slope is $\frac{1}{8}$. The given point is on the y-axis, so its y-coordinate tells you the y-intercept is 0. The equation is $y = \frac{1}{8}x + 0$ or $y = \frac{1}{8}x$.

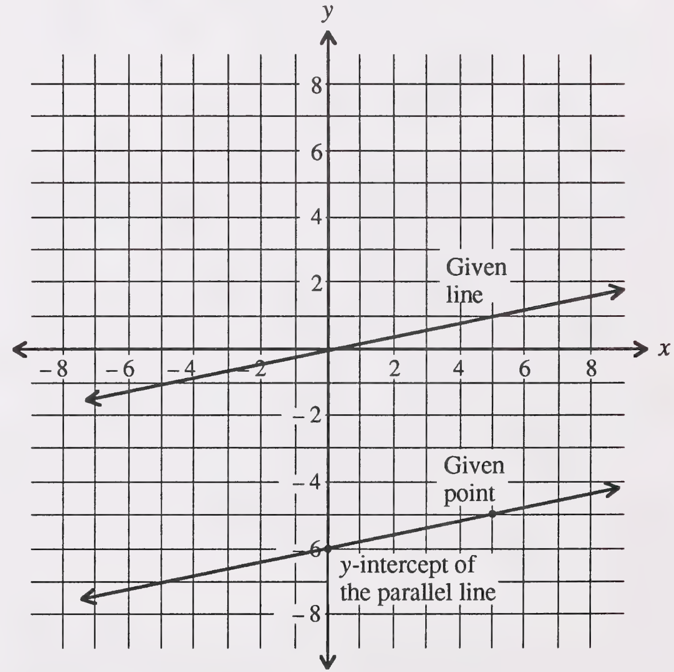


9. For the parallel line, the slope is $\frac{1}{5}$.

$$\begin{array}{c} \tfrac{1}{5} \\ \downarrow \\ y = mx + b \\ \uparrow \quad\quad \uparrow \\ -5 \quad\quad 5 \end{array}$$

$$-5 = \left(\frac{1}{5}\right)(5) + b$$
$$-5 = 1 + b$$
$$b = -6$$
$$y = \frac{1}{5}x - 6$$

The y-intercept of the parallel line is -6, so this line can be graphed by drawing a line through $(0, -6)$ and $(5, -5)$.

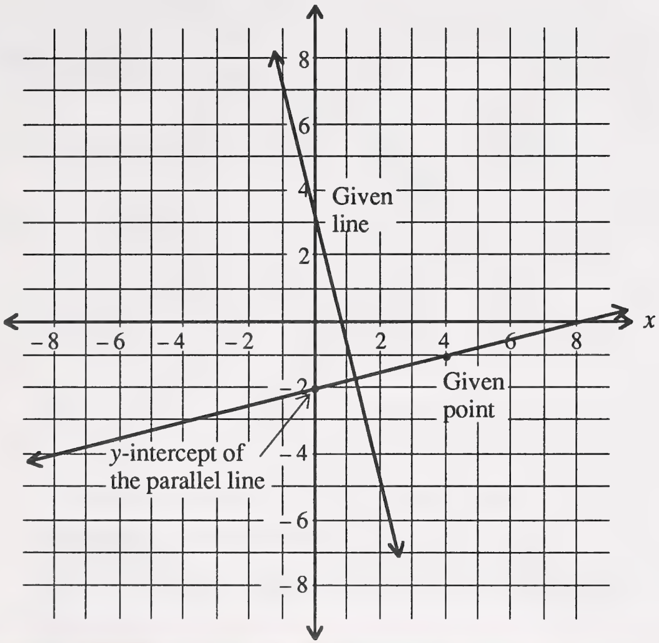


10. The slope of the given line is – 4. The slope of the perpendicular line is $\frac{-1}{(-4)} = \frac{1}{4}$.

$$\boxed{\begin{array}{c} \tfrac{1}{4} \\ \downarrow \\ y = mx + b \\ \uparrow \quad\quad \uparrow \\ -1 \quad\quad 4 \end{array}}$$

$$-1 = \left(\frac{1}{4}\right)(4) + b$$
$$-1 = 1 + b$$
$$b = -2$$
$$y = \frac{1}{4}x - 2$$

The y-intercept of the perpendicular line is – 2, so it runs through (0, – 2). This line can be graphed by drawing a line through (0, – 2) and the given point (4, – 1).

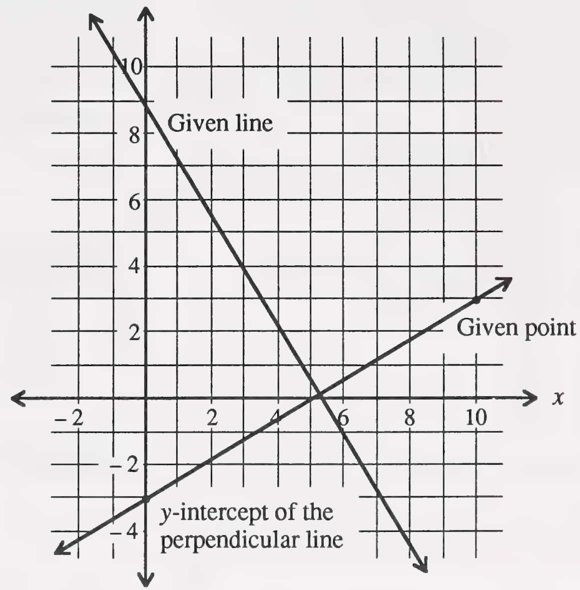


11. The slope of the given line is $-\frac{5}{3}$. The slope of the perpendicular line is $\frac{-1}{(-\frac{5}{3})} = \frac{3}{5}$.

$$\boxed{\begin{array}{c} \tfrac{3}{5} \\ \downarrow \\ y = mx + b \\ \uparrow \quad\quad \uparrow \\ 3 \quad\quad 10 \end{array}}$$

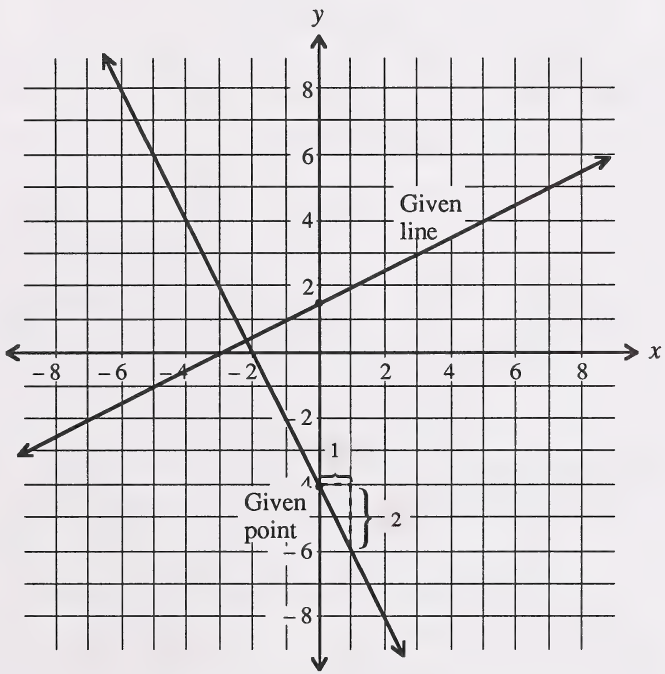
$$3 = \left(\frac{3}{5}\right)(10) + b$$
$$3 = 6 + b$$
$$b = -3$$
$$y = \frac{3}{5}x - 3$$

The y-intercept of the perpendicular line is – 3. The graph runs through (0, – 3) and the given point is (10, 3).



12. The slope of the given line is $\frac{1}{2}$, so the slope of the perpendicular line is $\frac{-1}{\left(\frac{1}{2}\right)} = -2$.

The point (0, – 4) is on the line and its y-intercept is – 4. The equation of the perpendicular line passing through (0, – 4) is $y = -2x - 4$. The graph is drawn using the slope and y-intercept.



13. You must first determine the slope of the given line.

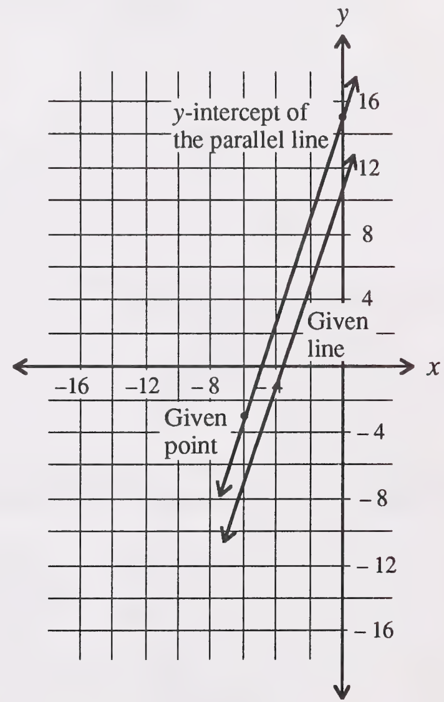
$$3x - y = -11$$
$$-y = -3x - 11$$
$$y = 3x + 11$$

The slope of the given line is 3, so the slope of the parallel line through (– 6, – 3) is 3.

$$\boxed{\begin{array}{c} \quad 3 \\ \quad \downarrow \\ y = mx + b \\ \uparrow \qquad \uparrow \\ -3 \quad -6 \end{array}}$$

$$-3 = 3(-6) + b$$
$$-3 = -18 + b$$
$$b = 15$$
$$y = 3x + 15$$

The y-intercept of the parallel line is +15. This line can be graphed by drawing a line through (0, 15) and the given point (– 6, – 3).



14. The given equation is put into the slope-intercept form to determine the slope of the given line.

$$\begin{aligned} 3x+4y&=10\\ 4y&=-3x+10\\ y&=-\frac{3}{4}x+2\frac{1}{2} \end{aligned}$$

The slope of the given line is $-\frac{3}{4}$. The slope of the perpendicular line through (1, 2) is $\frac{-1}{\left(-\frac{3}{4}\right)}=\frac{4}{3}$.

$$\boxed{\begin{array}{c} \tfrac{4}{3}\\ \downarrow\\ y=mx+b\\ \uparrow\quad\uparrow\\ 2\quad\ 1 \end{array}}$$

$$\begin{aligned} 2&=\left(\frac{4}{3}\right)(1)+b\\ \frac{6}{3}&=\frac{4}{3}+b\\ b&=\frac{2}{3}\\ y&=\frac{4}{3}x+\frac{2}{3} \end{aligned}$$

The equation of the perpendicular line through (1, 2) is

$$y=\frac{4}{3}x+\frac{2}{3}\ .$$

15. The given line is horizontal, so the parallel line through (7, 4) will be a horizontal line through (7, 4). Such a line has the equation $y = 4$.

16. The given line is horizontal, so the perpendicular line through (– 5, 3) will be a vertical line through (– 5, 3). The equation of such a line is $x = -5$.

Extra Help

1. $m_0 = 3$ and $b_0 = -2$ in $y = m_0x + b_0$.
   The equation is $y = 3x - 2$.

2. $(x_0, y_0) = (14, 22)$, so $x_0 = 14$ and $y_0 = 22$.

$$\begin{aligned} b_0 &= y_0 - m_0x\\ &= 22-2(14)\\ &= -6 \end{aligned}$$

   Thus, in $y = m_0x + b_0, m_0 = 2$ and $b_0 = -6$.
   The equation is $y = 2x - 6$.

3. The x-coordinates are the same for the given points, so the equation of the line is $x = 2$.

4. $$\begin{aligned} m_0 &= \frac{10-4}{4-2}\\ &= \frac{6}{2}\\ &= 3\\ b_0 &= y_1 - m_0x_1\\ &= 4-(3)(2)\\ &= -2 \end{aligned}$$

   So, in $y = m_0x + b_0, m_0 = 3$ and $b_0 = -2$.
   The equation is $y = 3x - 2$.

5. The given equation must first be put into the $y = mx + b$ form.

$$2x - y = -1$$
$$-y = -2x - 1$$
$$y = 2x + 1$$

The slope of the given line is 2, so the slope of the parallel line through (5, 14) is 2. In $y = m_0 x + b_0$, $m_0 = 2$ and

$$b_0 = y_0 - m_0 x_0$$
$$= 14 - 2(5)$$
$$= 14 - 10$$
$$= 4.$$

The equation of the parallel line is $y = 2x + 4$.

6. In the $y = mx + b$ form, the given relation is expressed by $y = 2x + 1$.
The equation of the perpendicular line is $y = m_0 x + b_0$, where

$$m_0 = \frac{-1}{2}$$
$$= -\frac{1}{2} \text{ and}$$

$$b_0 = y_0 - m_0 x_0$$
$$= 7 - \left(-\frac{1}{2}\right)(-6)$$
$$= 7 - 3$$
$$= 4.$$

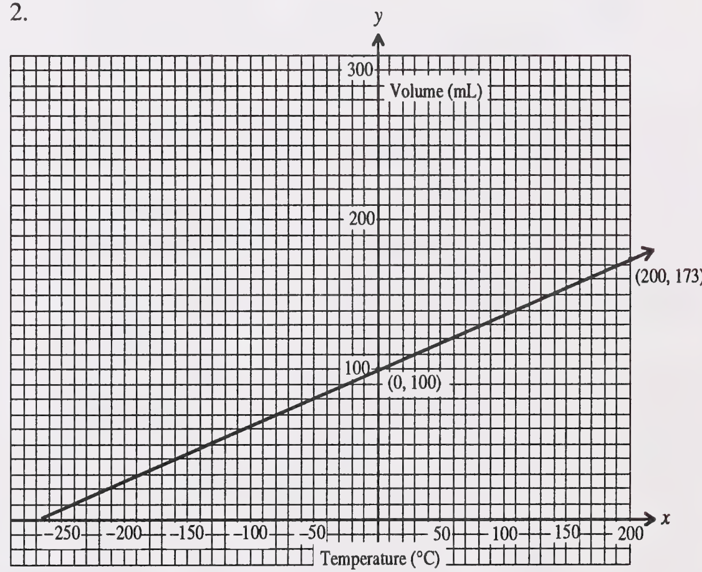
The equation is $y = -\frac{1}{2}x + 4$.

7. In $x = x_0$, $x_0 = 2$. The equation of the parallel line is $x = 2$.
8. In $x = x_0$, $x_0 = 2$. The equation of the perpendicular line is $x = 2$.
9. In $y = y_0$, $y_0 = 7$. The equation of the perpendicular line is $y = 7$.

Extensions

1. The points are (0, 100) and (200, 173). See the graph for question 2.

2.



3. Let $(x_1, y_1) = (0, 100)$ and $(x_2, y_2) = (200, 173)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{173 - 100}{200 - 0} \\ &= \frac{73}{200} \end{aligned}$$

The point (0, 100) is on the line, so the y-intercept is 100.

The equation of the line is $y = \frac{73}{200}x + 100$.

4. To calculate absolute zero from the equation, determine the x-intercept for the relation.

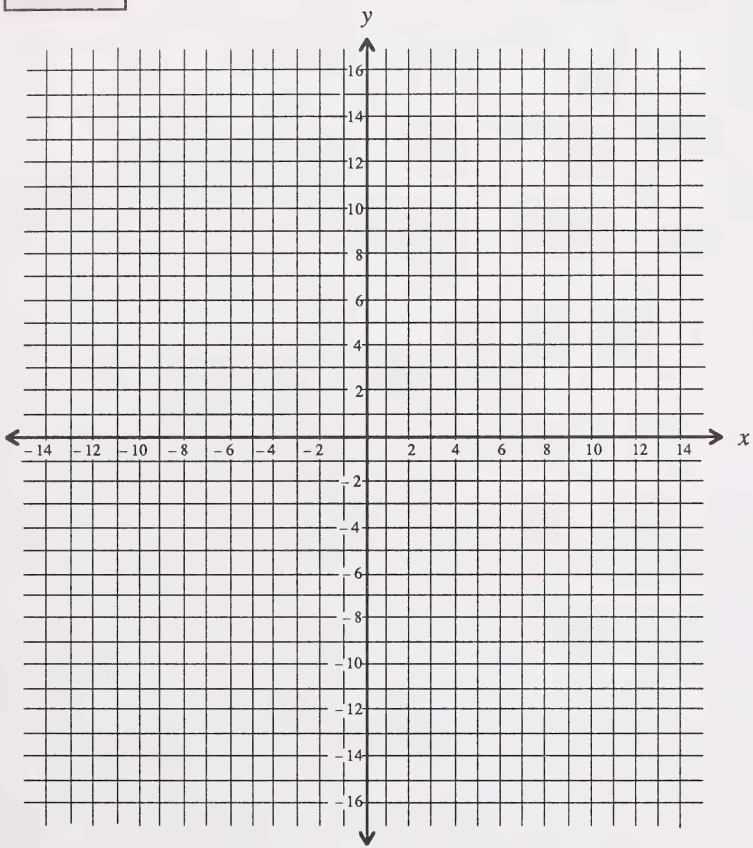
$$0 = \frac{73}{100}x + 100$$

$$\frac{73}{200}x = -100$$

$$x = -100 \times \left(\frac{200}{73}\right) = -273.8°\mathrm{C}$$

5. Absolute zero is – 273° C. The calculated value is within 1° C of the correct value.

Appendix B



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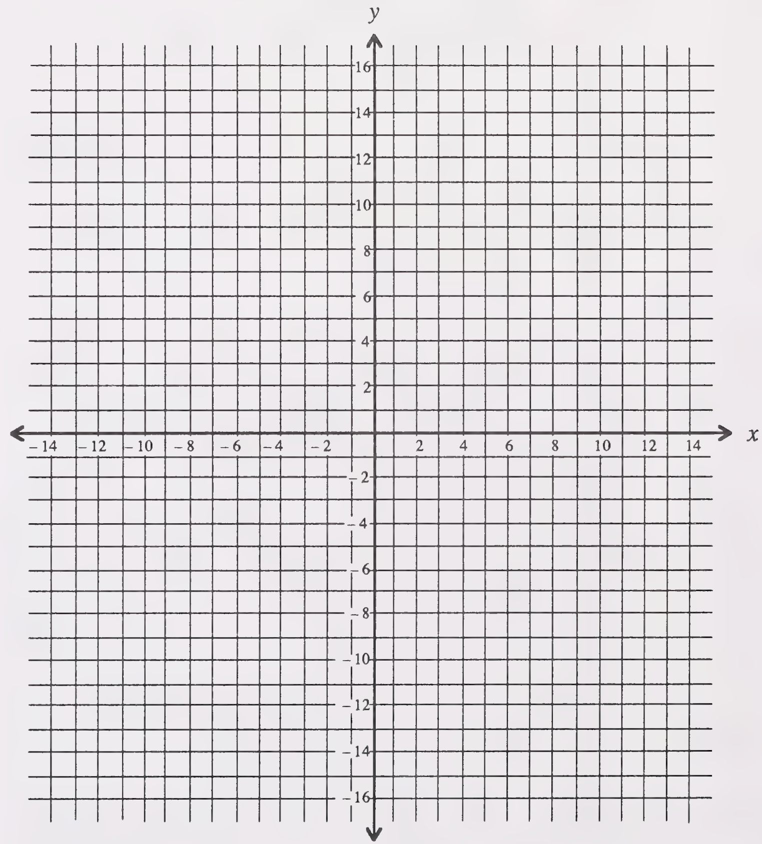

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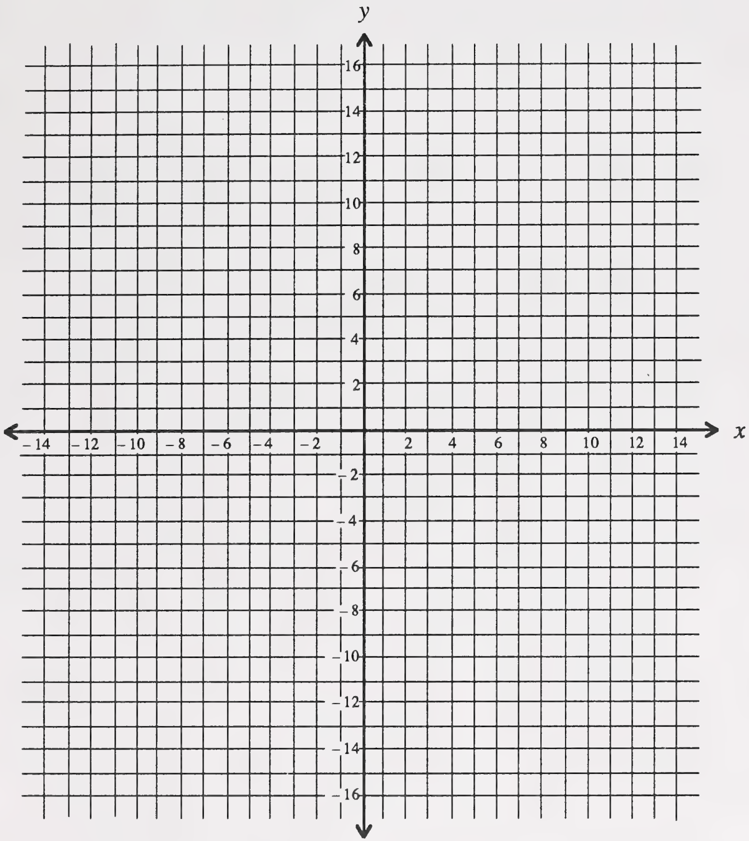

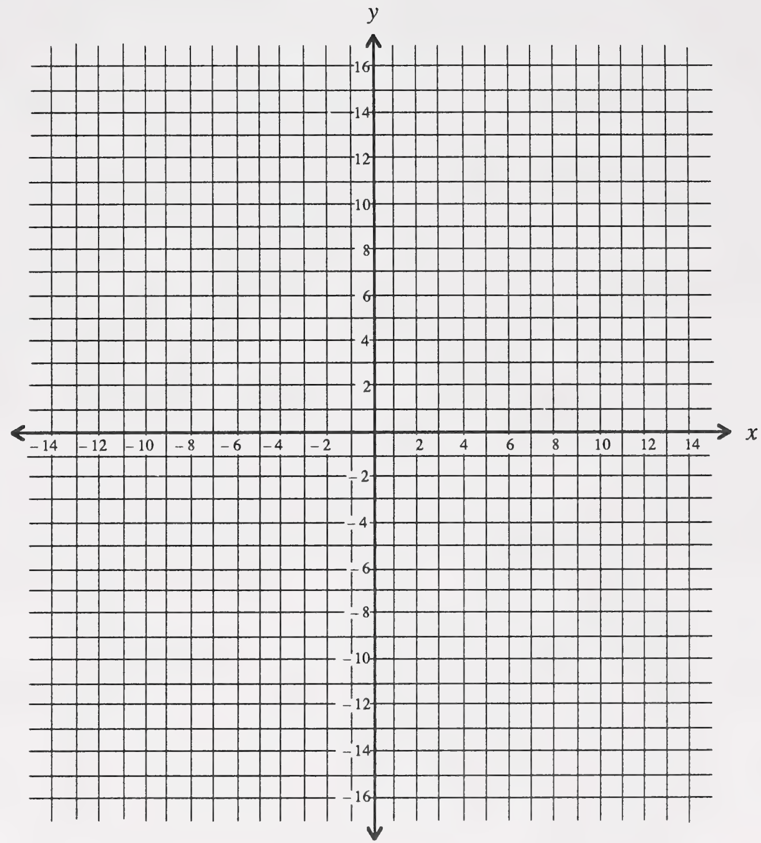

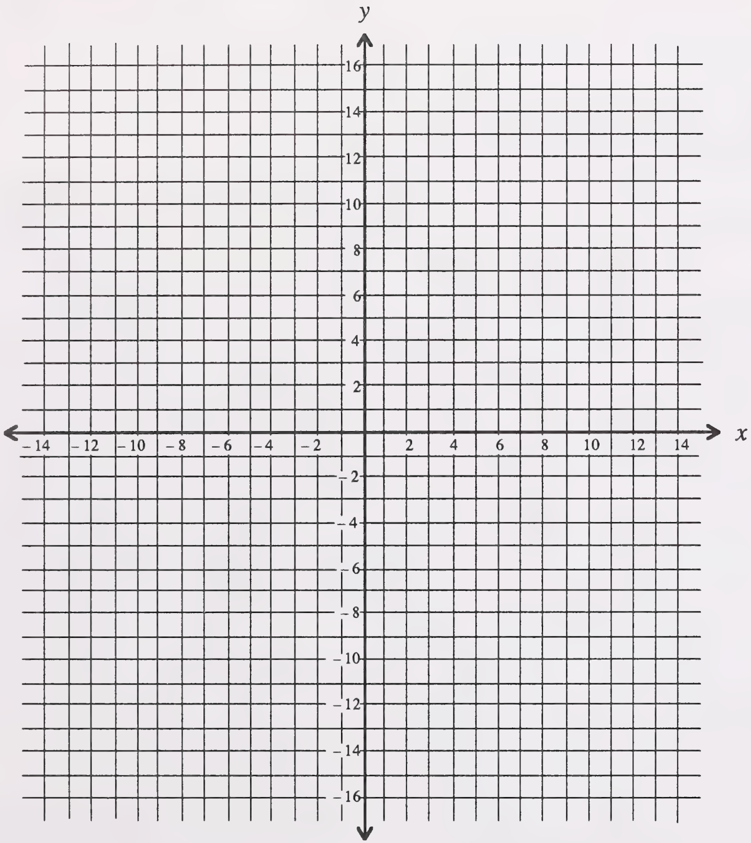
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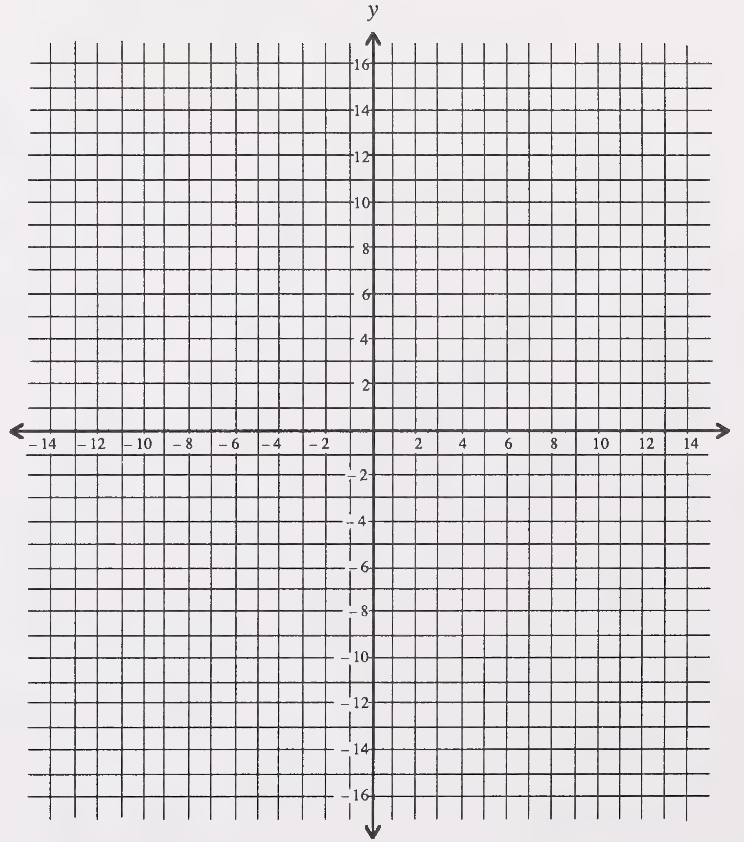
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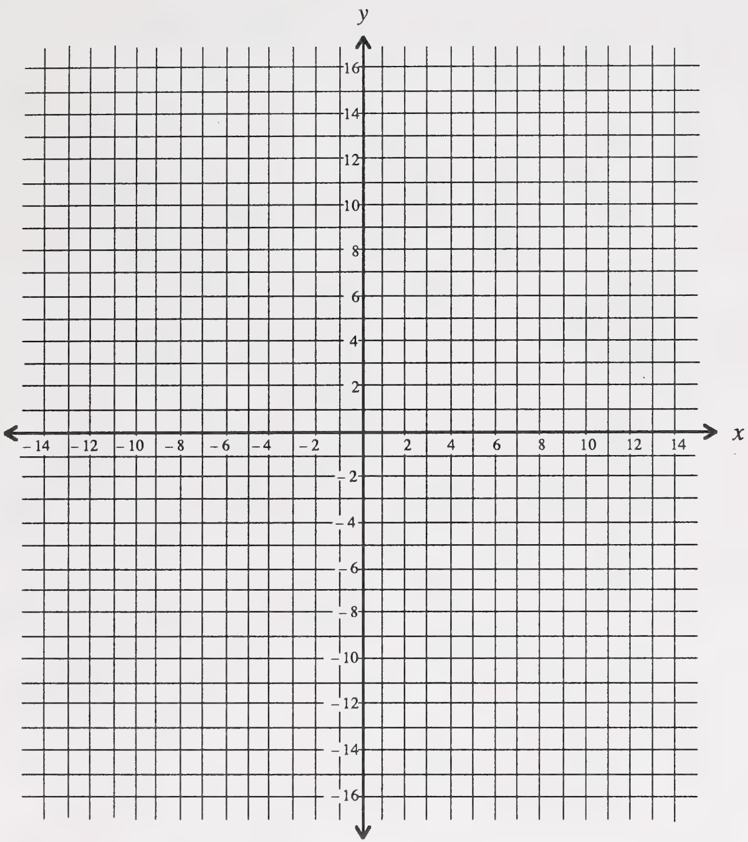
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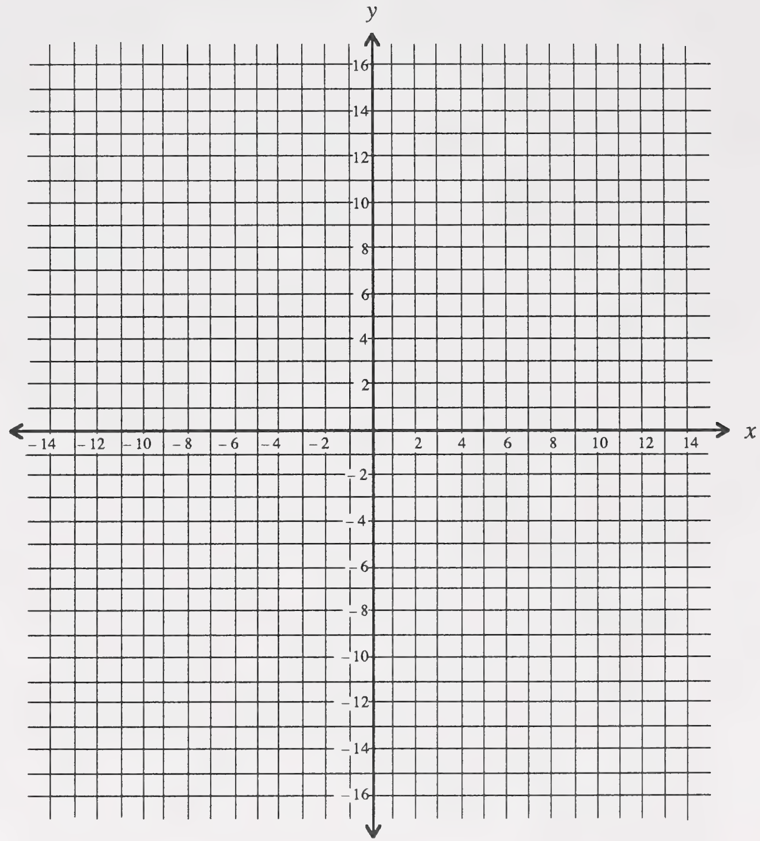
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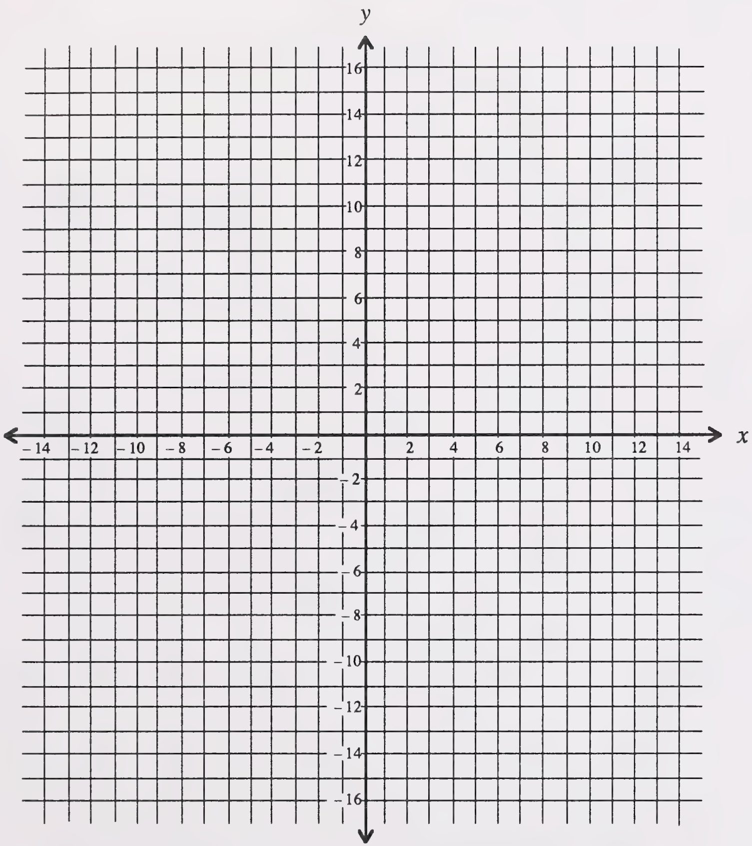

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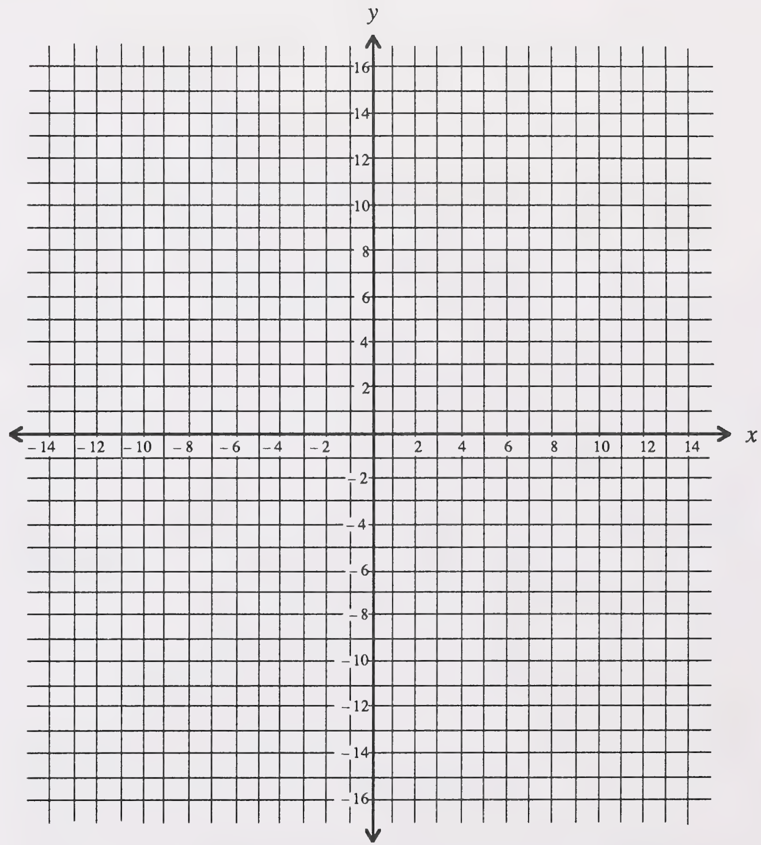

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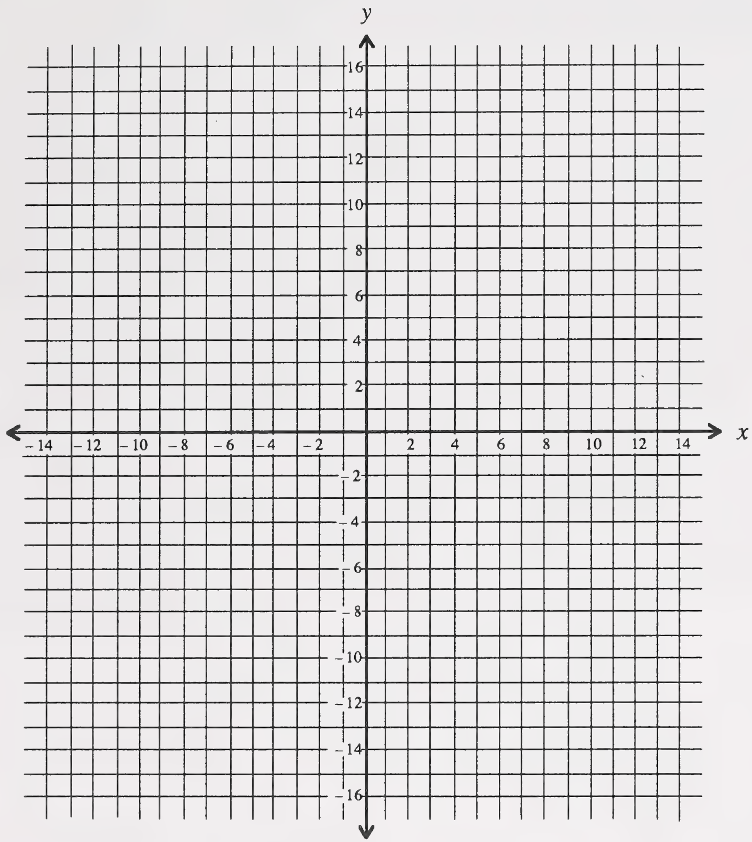
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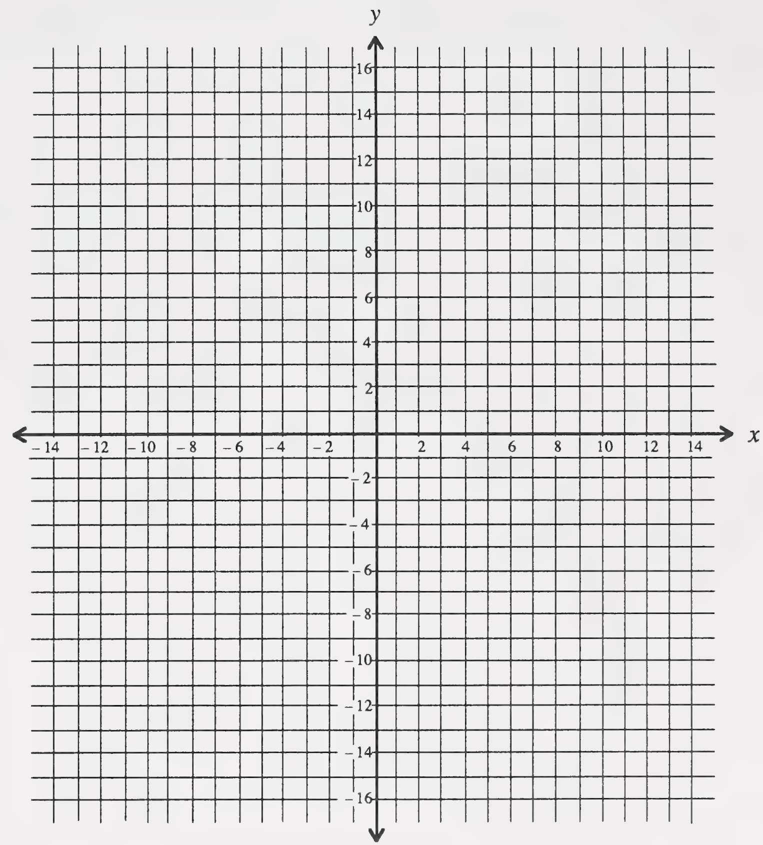
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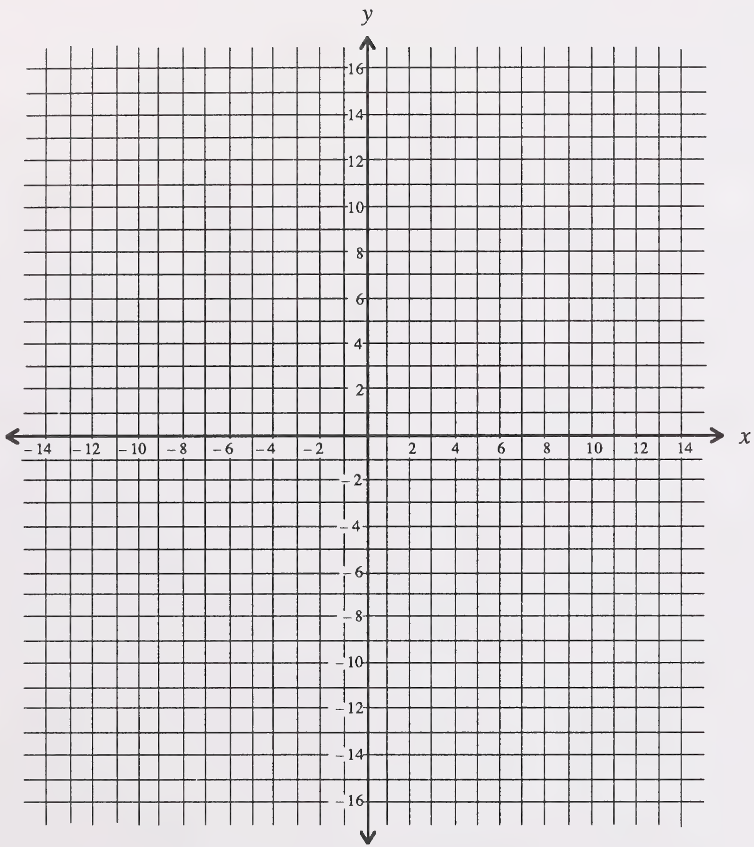

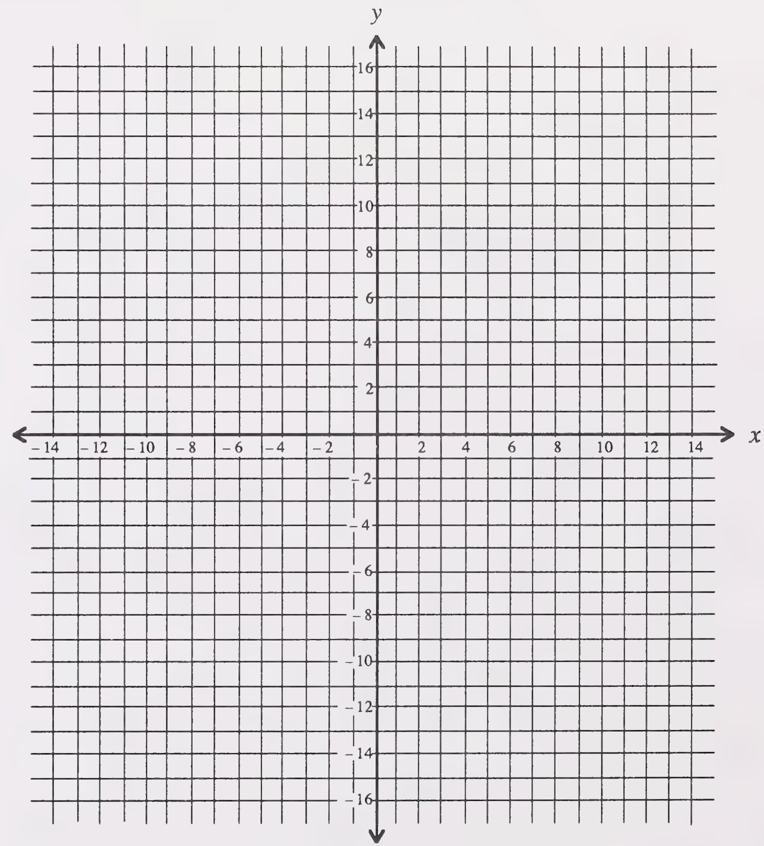

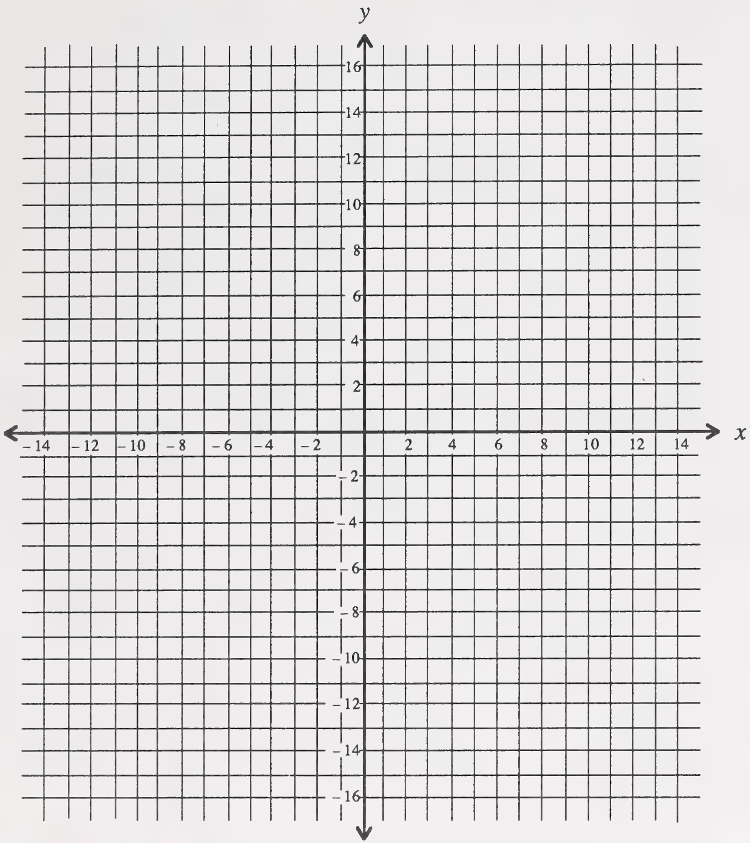

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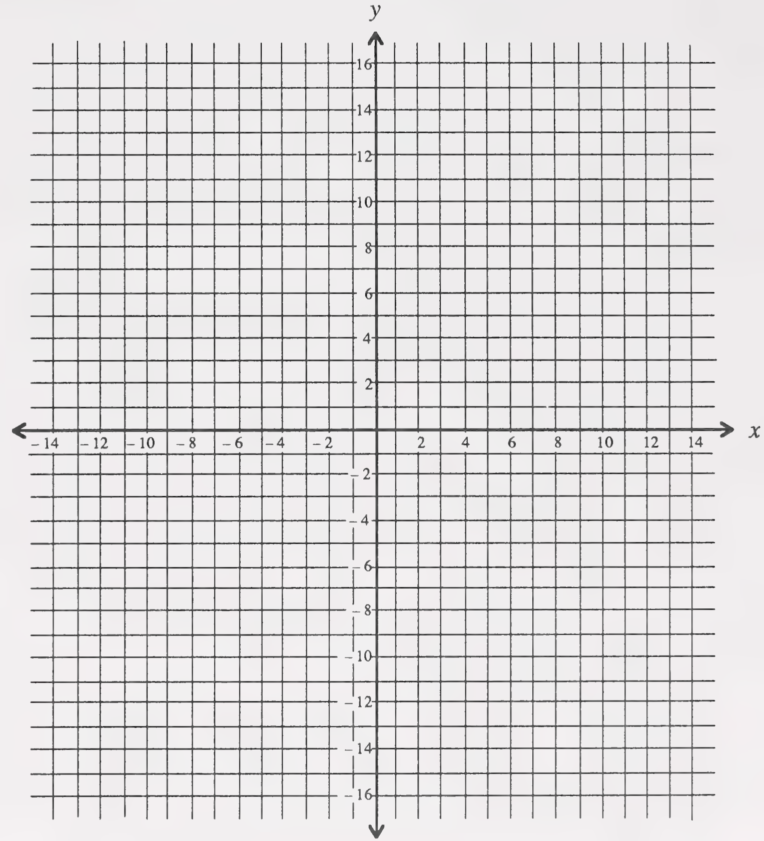

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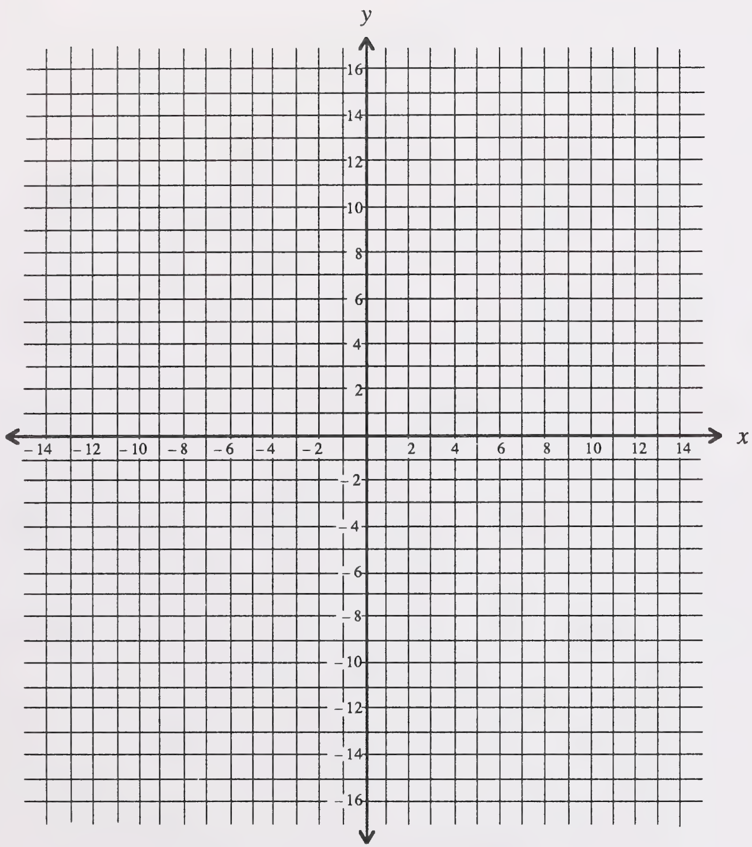

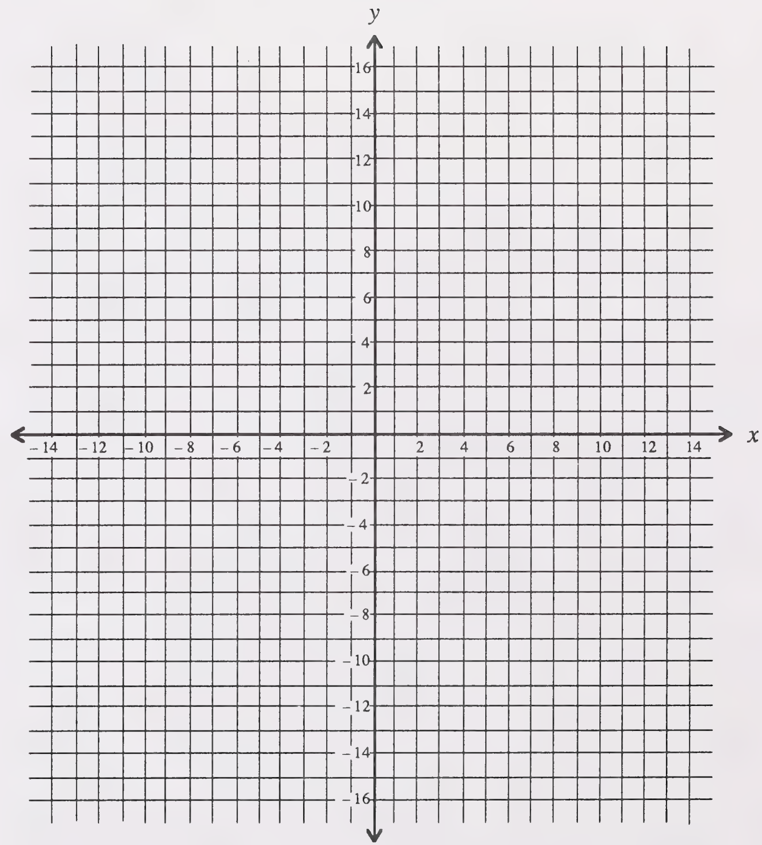

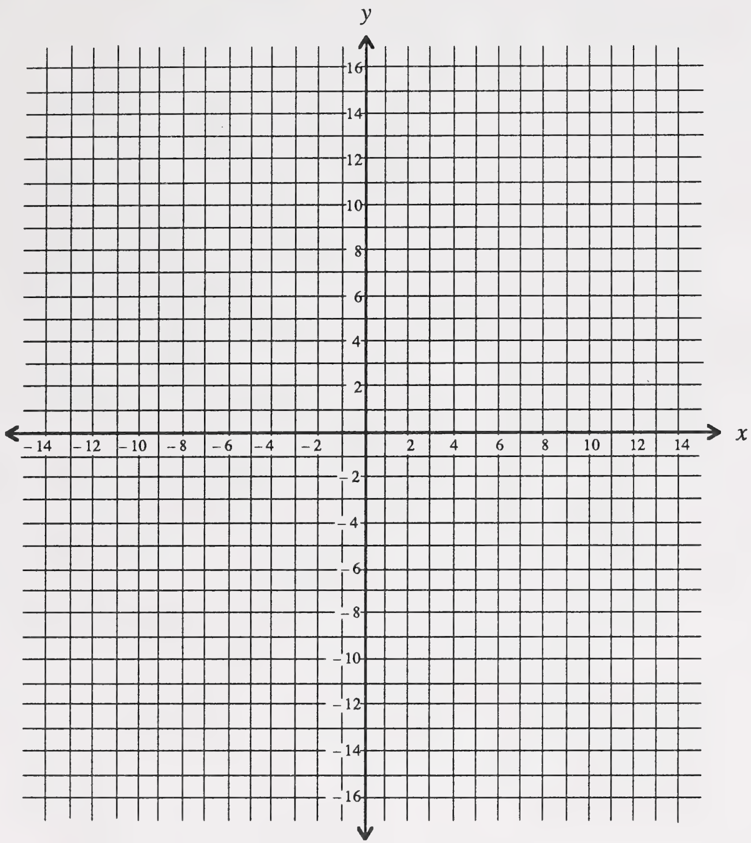
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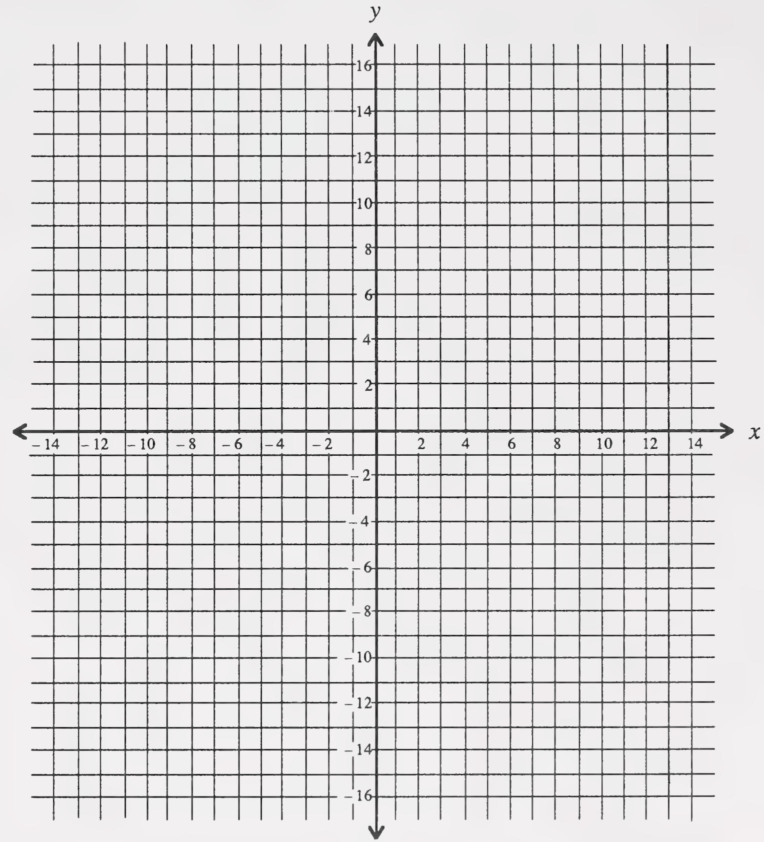
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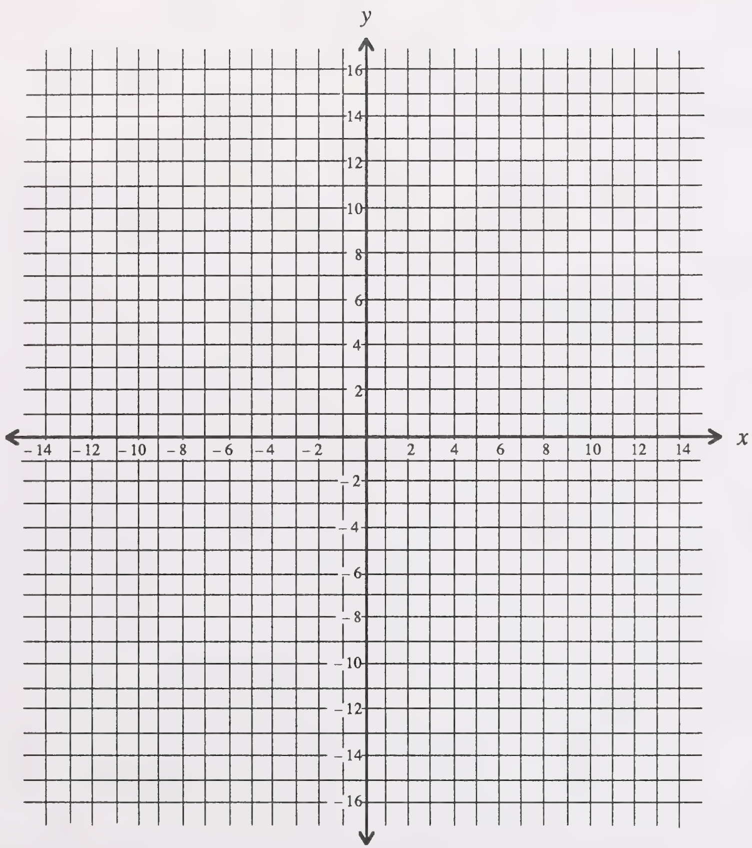

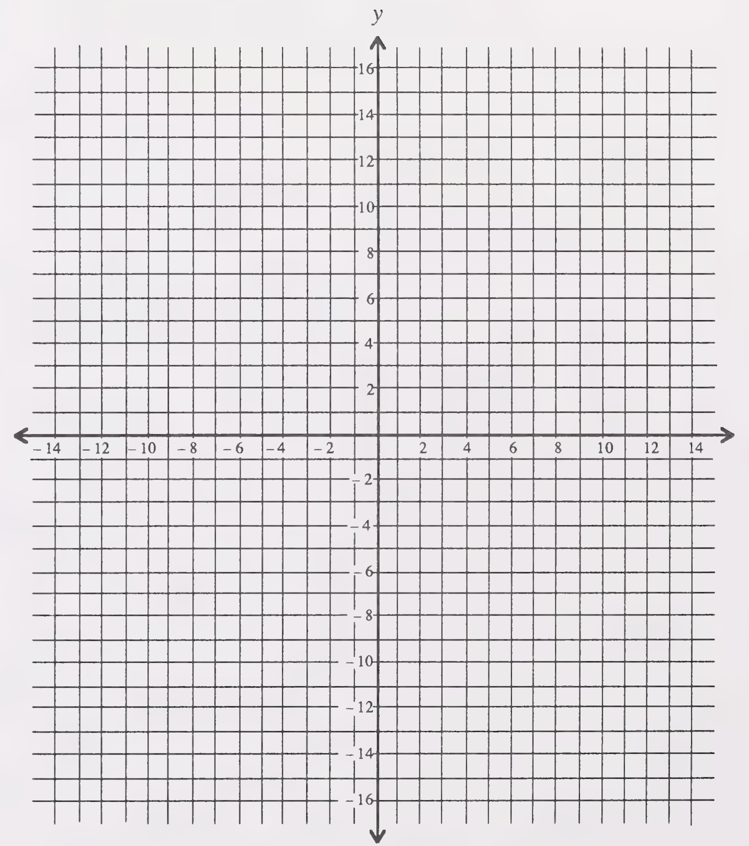

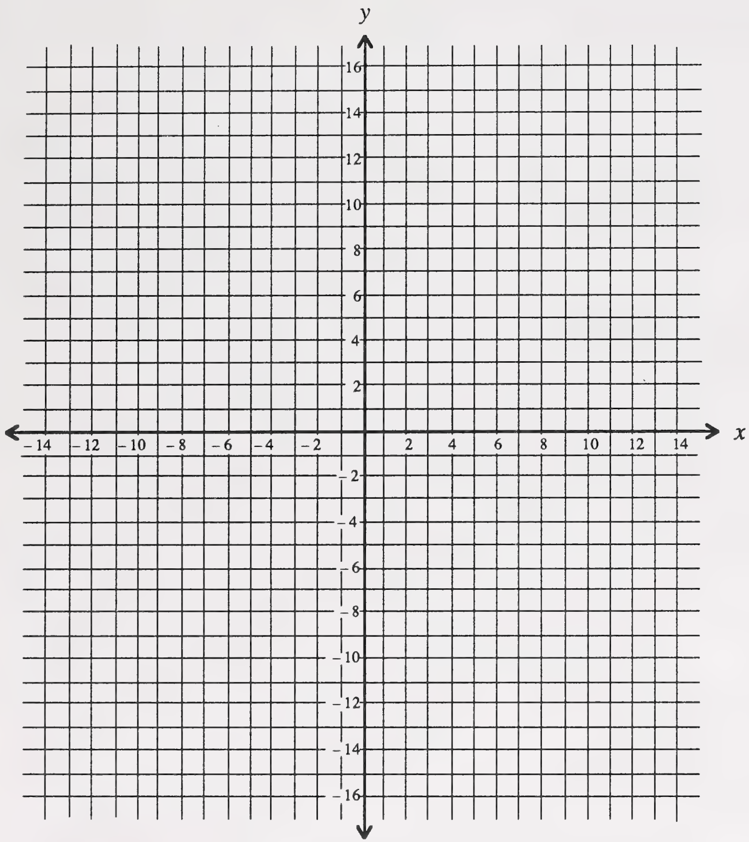

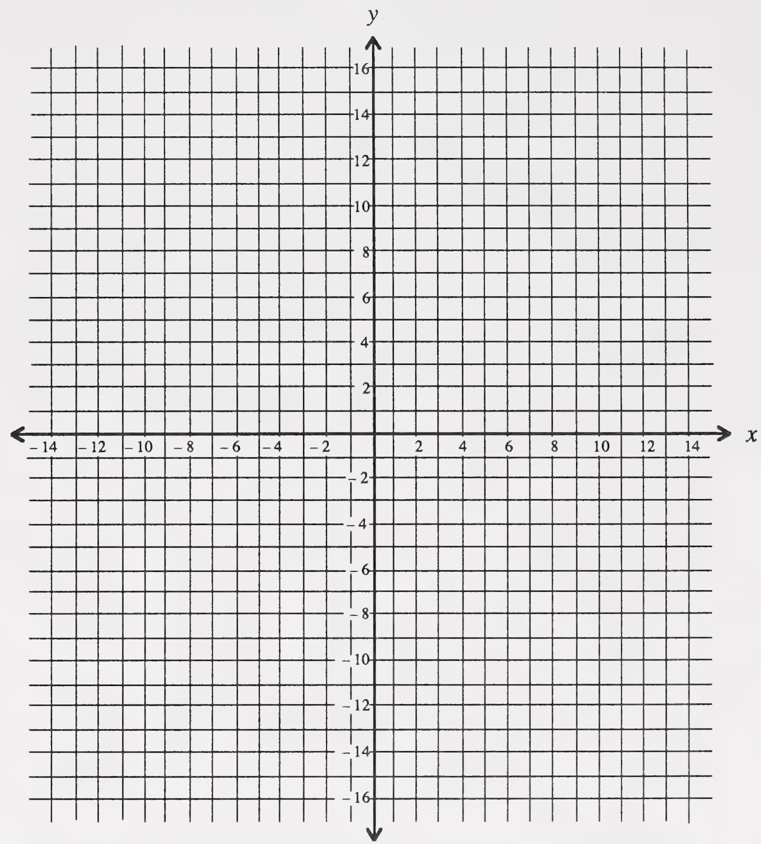

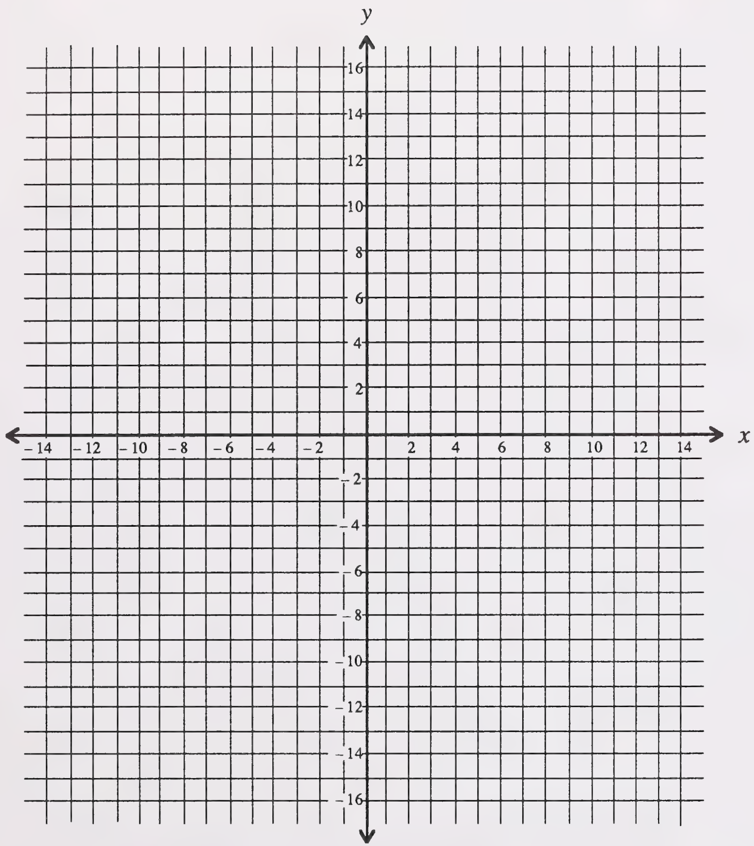
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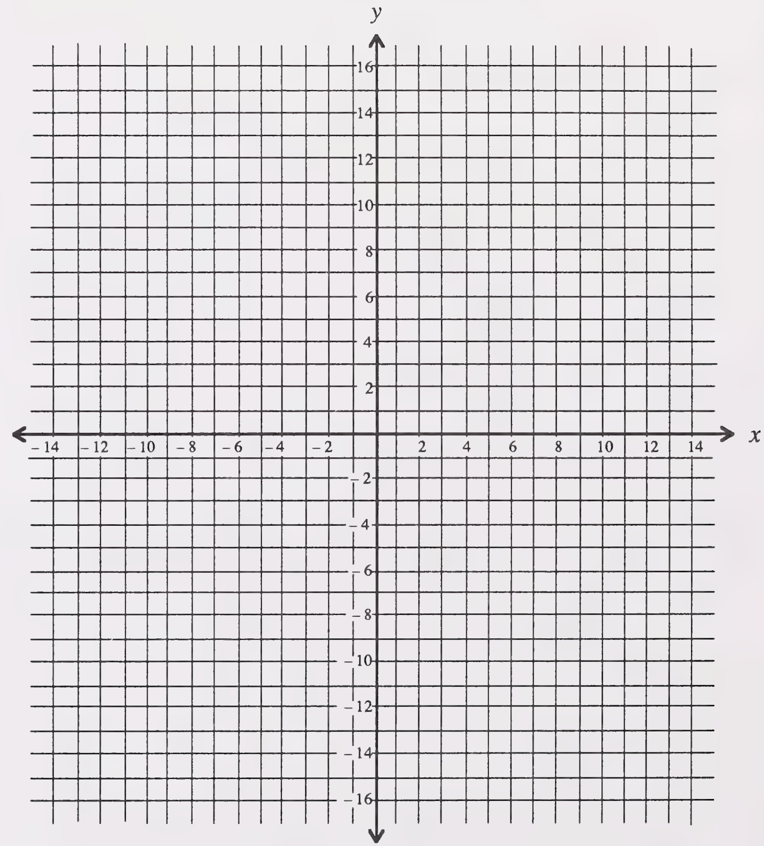
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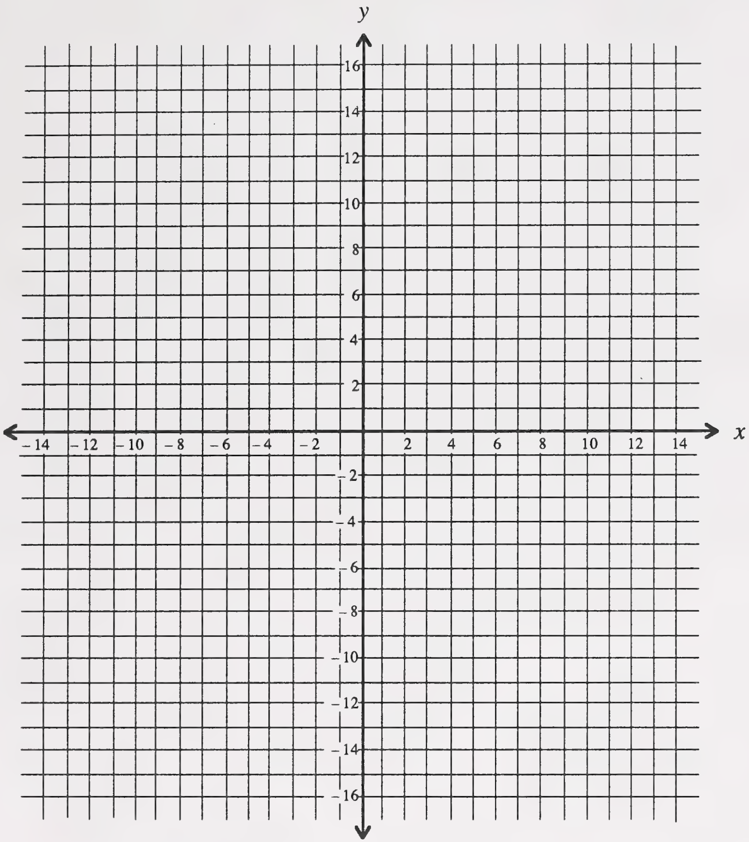

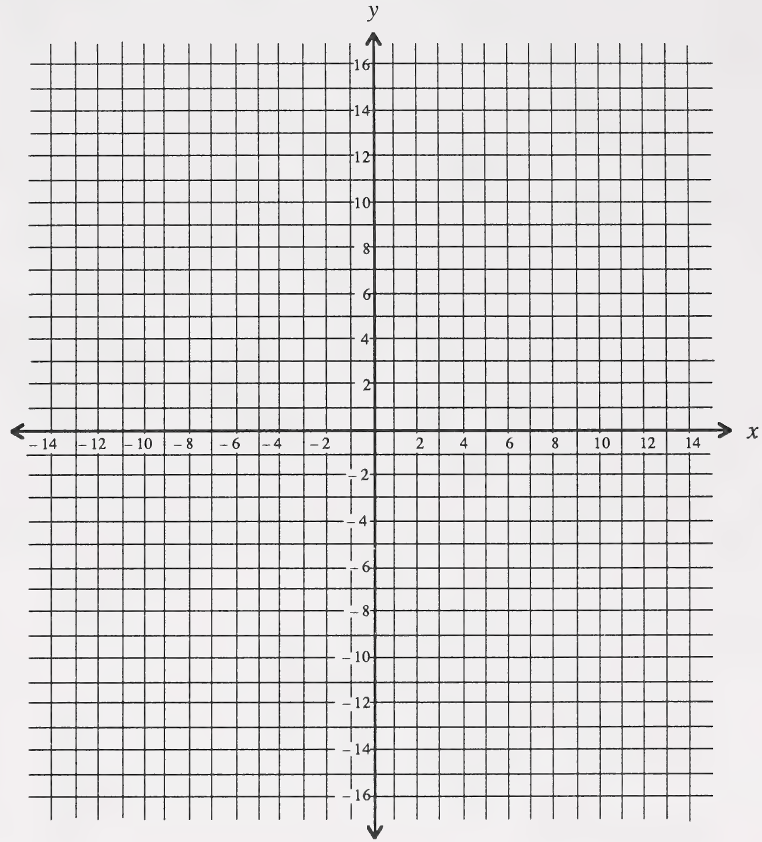

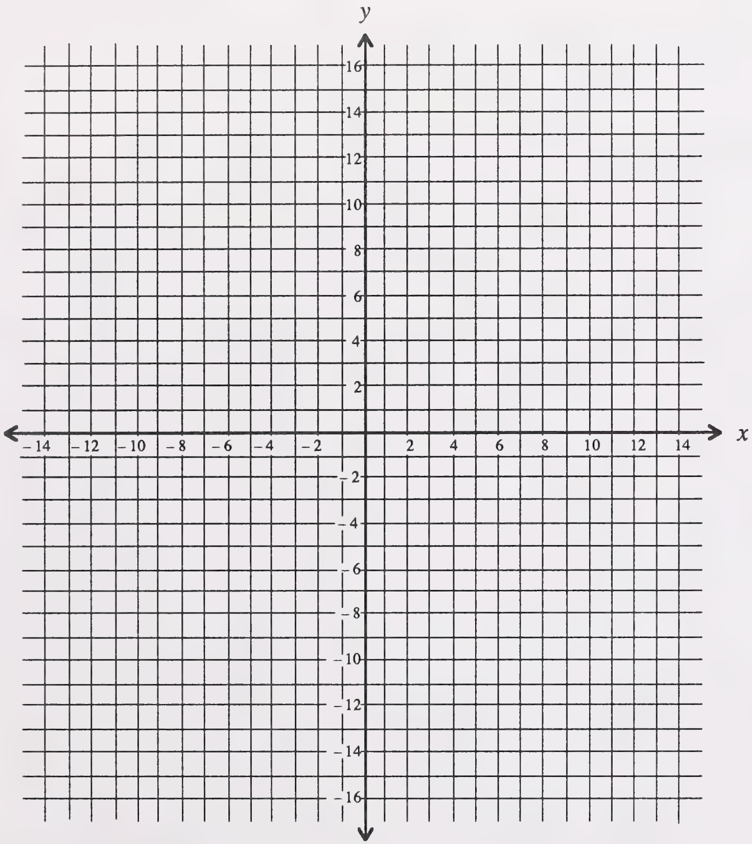
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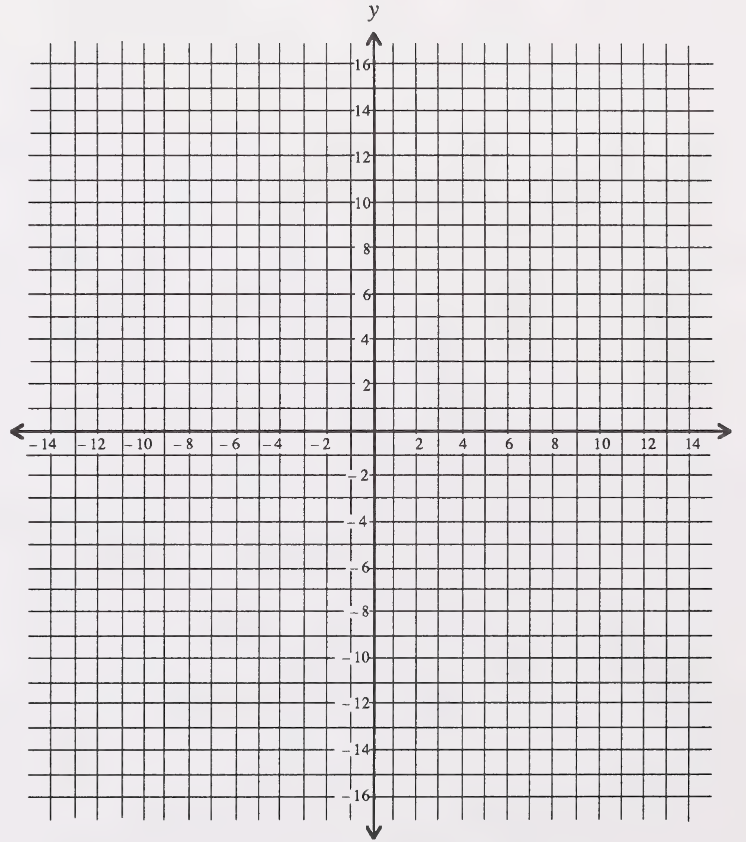
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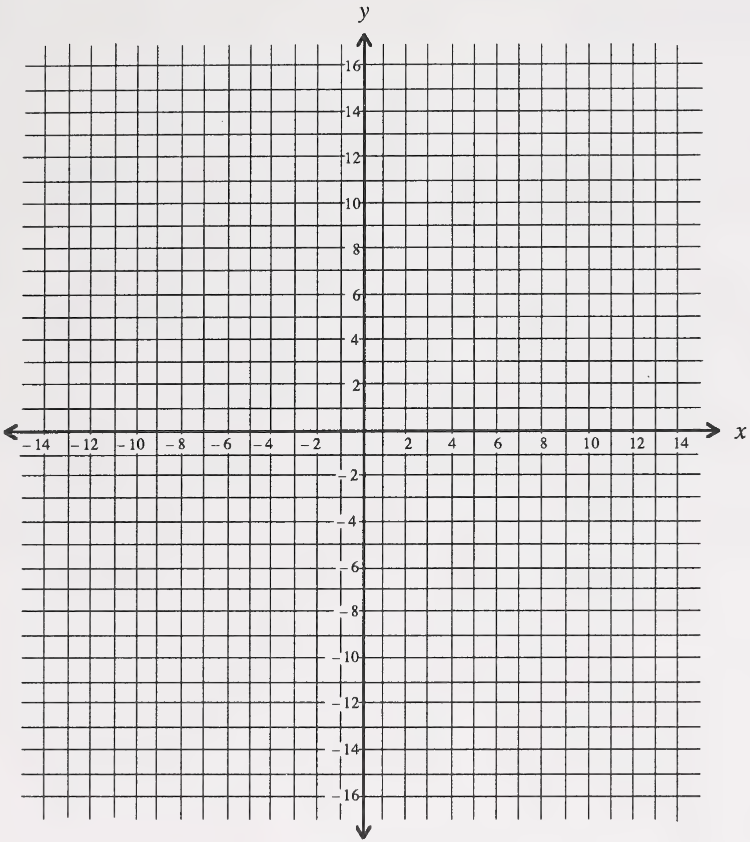
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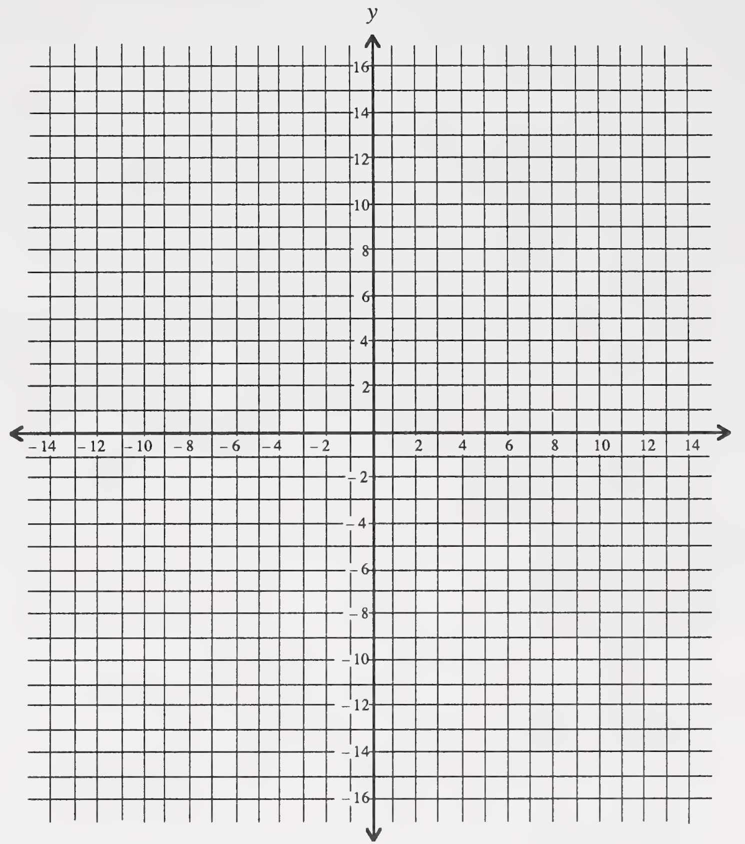
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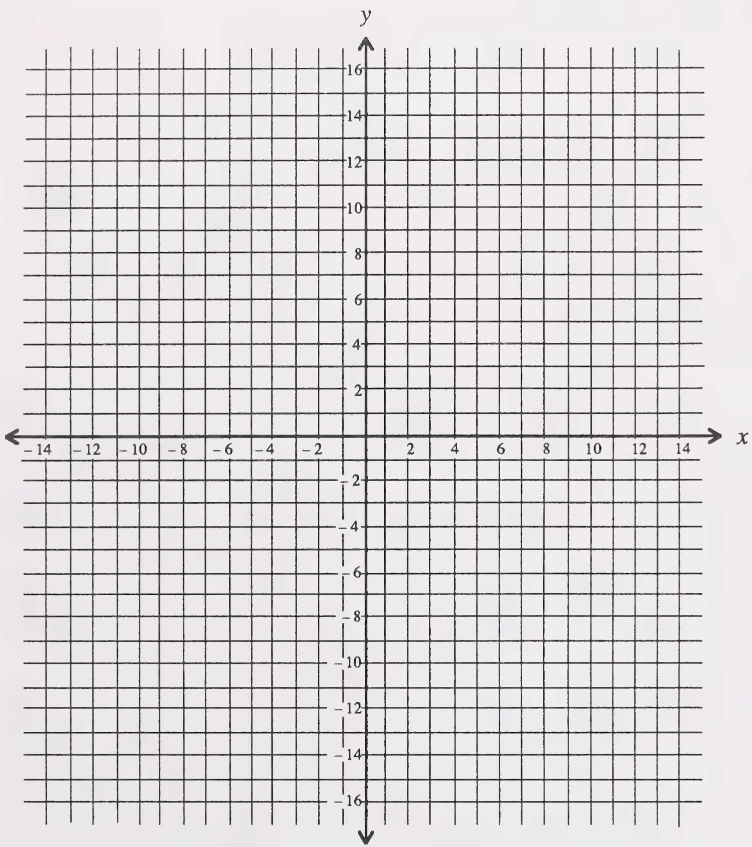

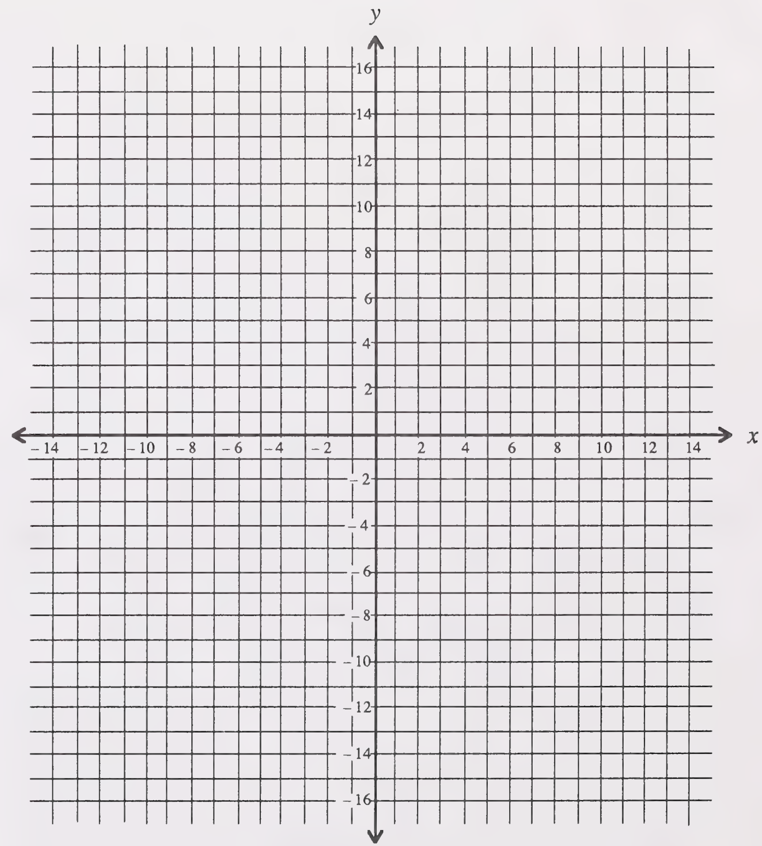

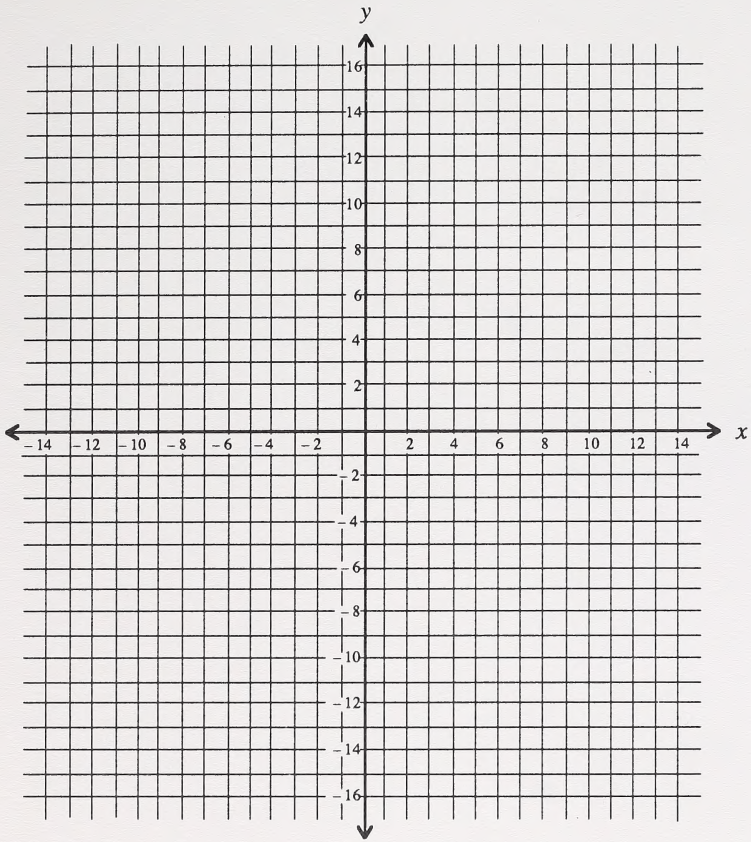
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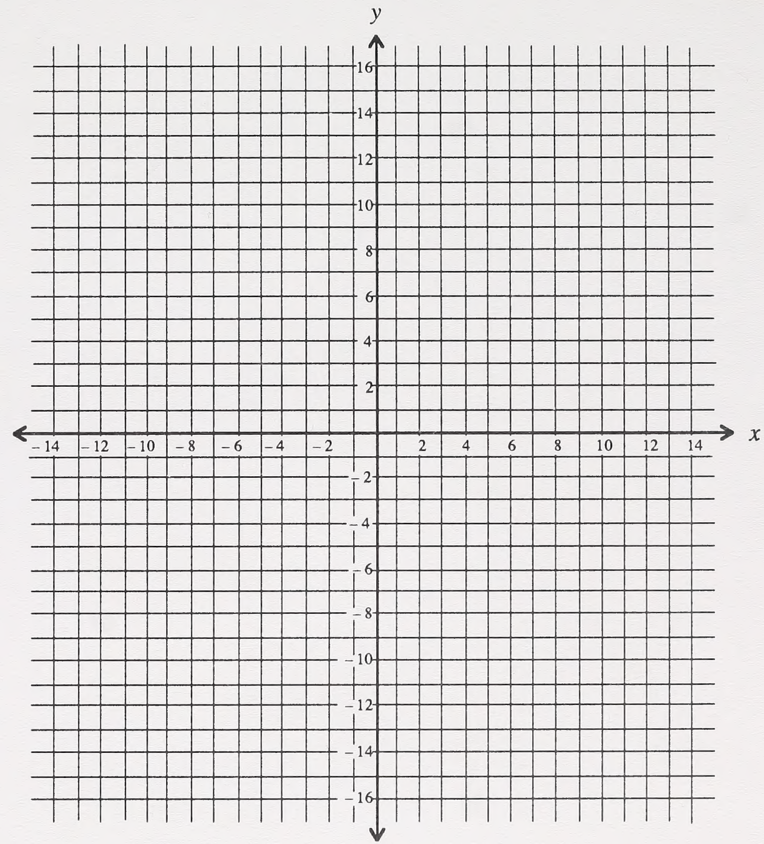
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